Circles

Degrees $\longleftrightarrow$ radians

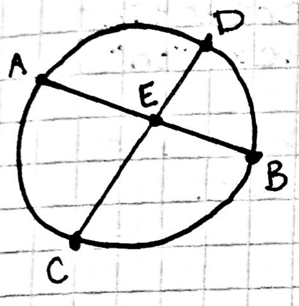
Degrees $\cdot \frac{\pi}{180^\circ}$ Radians $\cdot \frac{180^\circ}{\pi}$

Example

$120^\circ \cdot \frac{\pi}{180} = \frac{2\pi}{3}$ $\frac{9\pi}{4} \cdot \frac{180^\circ}{\pi} = \frac{1620\cancel{\pi}}{4\cancel{\pi}} = 405^\circ$

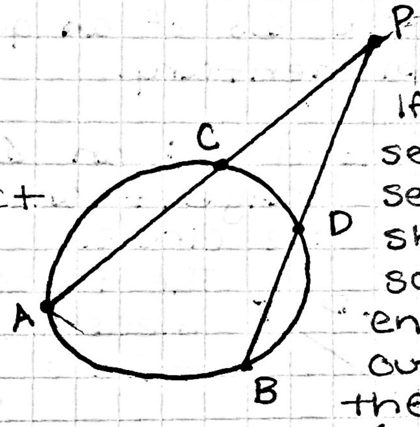
Circumference : $2\pi r$

Area : πr^2



Two chords intersect on the interior of a circle :

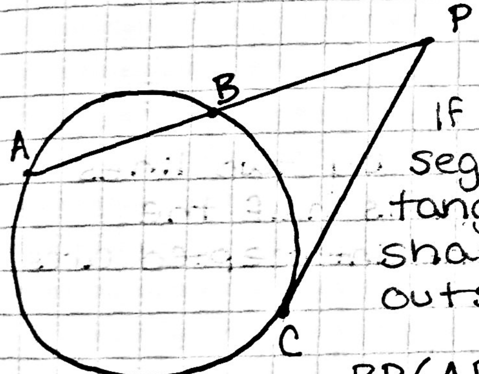
AE(EB) = DE(EC)



If two secant segments share the same endpoint outside of the circle:

CP(AP) = DP(BP)

(whole segment)(exterior) = (whole)(exterior)

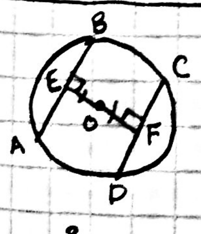


If a secant segment and a tangent segment share an endpoint outside the circle:

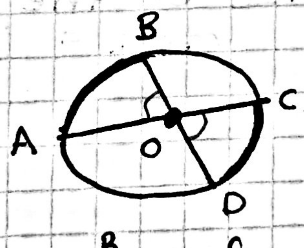
$BP(AP) = (CP)^2$

$(\text{whole segment})(\text{exterior}) = (\text{tangent})^2$

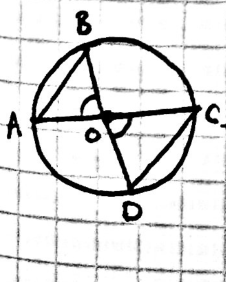
Chords & Arcs



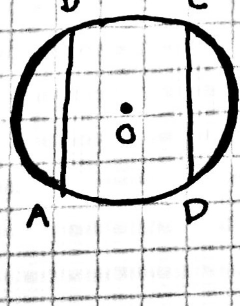
If $\overline{OE} \cong \overline{OF}$ then $\overline{AB} \cong \overline{DC}$



If $\angle AOB \cong \angle COD$ then $\overset{\frown}{AD} \cong \overset{\frown}{CD}$

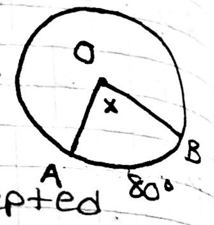


If $\angle AOB \cong \angle COB$ then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

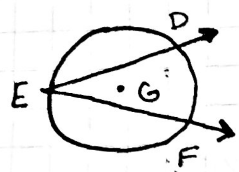


If $\overline{AB} \cong \overline{CD}$ then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

Central $\angle$: is an $\angle$ formed by two intersecting radii such that its vertex is the center



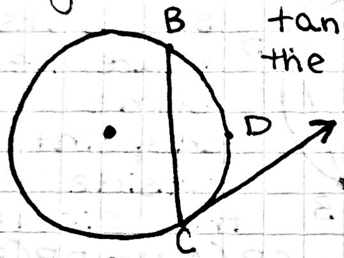
Inscribed $\angle$: is an $\angle$ with its vertex "on" the circle, formed by two intersecting chords



The measure of an inscribed $\angle$ is half the measure of its intercepted arc. $\angle DEF = \frac{1}{2} m\overset{\frown}{DF}$

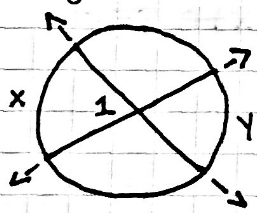
The opposite $\angle$s of a quadrilateral inscribed in a circle are supplementary.

Tangent Chord $\angle$: an $\angle$ formed by an intersecting tangent and chord has its vertex "on" the circle.

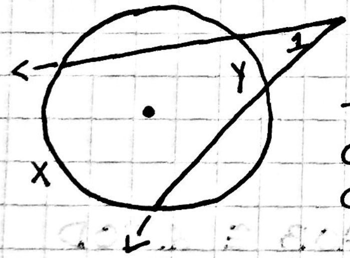


The tangent chord angle is half the measure of the intercepted arc.
$m\angle C = \frac{1}{2} m\overset{\frown}{BDC}$

Angle Measures



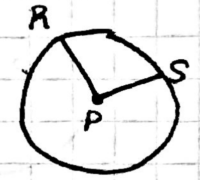
The measure of an $\angle$ formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.
$m\angle 1 = \frac{1}{2}(x+y)$



The measure of an $\angle$ formed by two lines that intersect outside a circle is $\frac{1}{2}$ the difference of the measures of the intercept arcs. $m\angle 1 = \frac{1}{2}(x-y)$

Area of Sector

Sector of a Circle: The region bound by an arc of the circle and the two radii that meet at the endpoints of the arc.



Sector RPS

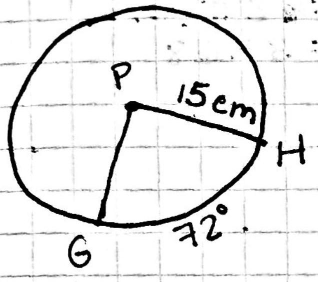
Area of Sector (Radians)

$A = \pi r^2 \cdot \frac{\theta}{2\pi}$ ← in radians

Area of Sector (Degrees)

$A = \pi r^2 \cdot \frac{\theta}{360}$ ← in degrees

Example 1

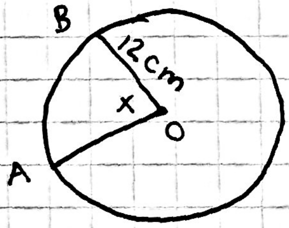


$A = (3.14)(15)^2\left(\frac{72}{360}\right)$

$= 141.3\text{cm}^2$ or $45\pi \text{ cm}^2$

Example 2

The area of sector AOB is $28\pi \text{ cm}^2$. Find the measureme[nt] of the central ∠.



$28\pi = \pi(12)^2\left(\frac{x}{360}\right)$

$87.92 = \frac{452.16x}{360}$

$31651.2 = 452.16x$

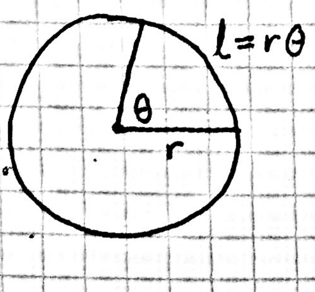
$x = 70°$

Arc length

Arc Length (in degrees): Convert the degrees to radians, and then use the radian formula!

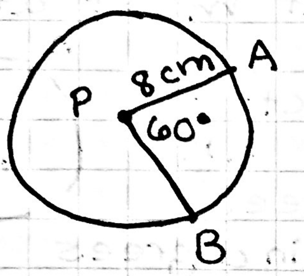
degrees → $\theta \cdot \frac{\pi}{180}$

The formula for arc length in radians is $l = r\theta$, where l is the arc length, r is the radius and θ is the measure of the ∠ subtended by the arc, in radians



Examples

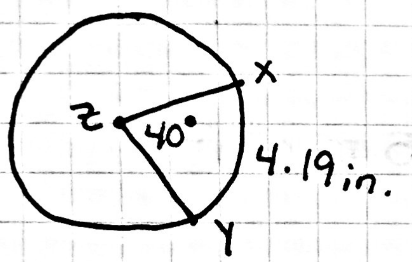
1) arc length of $\overset{\frown}{AB}$



$$60 \cdot \frac{\pi}{180} = \frac{60\pi}{180} = \frac{\pi}{3}$$

$$l = 8\left(\frac{\pi}{3}\right) = 8.37\text{ cm}$$

2) circumference of $\odot Z$

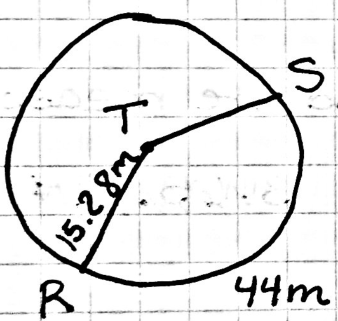


$$40 \cdot \frac{\pi}{180} = \frac{40\pi}{180} = \frac{2\pi}{9}$$

$$4.19 = \frac{2\pi}{9}(r) \qquad r = 6$$

$$C = 2\pi(6)$$
$$= 37.68$$

3) $m\overset{\frown}{RS}$



$$44 = \theta(15.28)$$

$\theta = 2.88 \leftarrow$ in radians

$$2.88 \cdot \frac{180}{\pi} = 165°$$