

Exponential Growth

Money in banks and populations often increase by a percentage rate over time.

Exponential Growth: $y = a(1+r)^t$

$$y = a(1+r)^t$$

↑ total ↑ initial amount ↑ rate (decimal) ↑ time

Example 1: \$1500 is deposited in a savings account paying an annual yield of 6%.

a) What is the percent written as a decimal? .06

b) Write an exponential growth equation.

$$y = 1500(1 + .06)^t$$

c) If the account is left alone, how much money will be in the account at the end of 10 years?

$$y = 1500(1 + .06)^{10}$$
$$= \$2686.27$$

Example 2) Jaime invests \$100 at an annual yield of 5%.

a) What is the percent written as a decimal? .05

b) Write an exponential growth equation.

$$y = 100(1 + .05)^t$$

c) If the account is left alone, how much money will be in the account at the end of 10 years?

$$y = 100(1 + .05)^{10}$$
$$= 162.89$$

Example 3: A home is worth \$125,000. It is appreciating at a rate of 3% per year.

a) What is the percent written as a decimal? .03

b) Write an exponential growth equation.

$$y = 125,000(1 + .03)^t$$

c) How much money will the house be worth in 5 years?

$$y = 125,000(1 + .03)^5$$
$$= 144909.26$$

Exponential Decay

Automobiles and manufactured goods often decrease in value over time.

Exponential Decay: $y = a(1 - r)^t$

$$y = a(1 - r)^t$$

final amount y
initial amount a
rate (decimal) r
time t

Example 1

In 2010, a new Firebird Trans Am cost \$15,798. Suppose the car depreciates 13% each year.

- a) What is the percent written as a decimal? .13
- b) Write an exponential decay equation.

$$y = 15,798(1 - .13)^t$$

- c) How much money will the car be worth in 5 years?

$$y = 15,798(1 - .13)^5$$
$$= \$7874.05$$

- d) How much money will the car be worth in 10 years?

$$y = 15,798(1 - .13)^{10}$$
$$= \$3924.59$$

Example 2:

Suppose a boat purchased for \$28,000 depreciates by 10% each year.

- a) What is the percent written as a decimal? .10
- b) Write an exponential decay equation.

$$y = 28000(1 - .10)^t$$

- c) How much money will the boat be worth in 2 years?

$$y = 28000(1 - .10)^2$$
$$= \$22680$$

- d) How much money will the car be worth in 5 years?

$$y = 28000(1 - .10)^5$$
$$= \$16533.72$$

Compound Interest

What is Compound Interest?

If you walk into a bank and open up a savings account you will earn interest on the money you deposit in the bank. If the interest is calculated once a year then the interest is called "simple interest". If the interest is calculated more than once per year, then it is called "compound interest".

Compound Interest Formula

The mathematical formula for calculating compound interest depends on several factors. These factors include the amount of money deposited called the principal, the annual interest rate (in decimal form), the number of times the money is compounded per year, and the number of years the money is left in the bank. These factors lead to the formula

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

A = future value of the deposit
 P = principal or amount of money deposited
 r = annual interest rate (in decimal form)
 n = number of times compounded per year
 t = time in years.

Interest can be compounded many different ways. You need to be on the lookout for some keywords to tell you what value to use for n in the formula.

Annually/Yearly:	<u>1</u>	Semi - annually:	<u>2</u>
Quarterly:	<u>4</u>	Monthly:	<u>12</u>
Weekly:	<u>52</u>	Daily:	<u>365</u>

Example 1: If you deposit \$4,000 into an account paying 6% annual interest compounded quarterly, how much money will be in the account after 5 years?

$$A = 4000 \left(1 + \frac{.06}{4} \right)^{4(5)}$$

$$= \$5387.42$$

Example 2: If you borrowed \$6,500 and pay 8% annual interest compounded monthly, how much money will you have paid back after 10 years?

$$A = 6500 \left(1 + \frac{.08}{12} \right)^{12(10)}$$

$$= 2912.83$$

Example 3: How much money would you need to deposit today at 9% annual interest compounded weekly to have \$12,000 in the account after 6 years?

$$12000 = P \left(1 + \frac{.09}{52} \right)^{52(6)}$$

$$\frac{12000}{1.7152} = \frac{1.7152 P}{1.7152}$$

$$P = 6996.24$$

Day 8 Practice Worksheet

1) There were 220 sports radio stations in the year 2000. The number of radio stations has since increased by approximately 14.3% per year.

- a) What is the percent written as a decimal? .143
- b) Write an exponential equation. $y = 220(1 + .143)^t$
- c) What will be the value of t in the year 2015?
 $t = 15$
- d) If the trend continues, how many sports radio stations will there be in 2015?
 $y = 1633.49$

2) Marlene invested \$500 in a savings account with an annual interest rate of 2.5%.

- a) What is the percent written as a decimal? .025
- b) Write an exponential equation. $y = 500(1 + .025)^t$
- c) How much money will Marlene have after 20 years?
 $y = 500(1 + .025)^{20}$
 $= 8819.31$

4) A new car costs \$18,000. It is expected to depreciate (decrease in value) at an average rate of 12% per year.

- a) What is the percent written as a decimal? .12
- b) Write an exponential equation. $y = 18000(1 - .12)^t$
- c) Find the value of the car in 8 years. $y = 18000(1 - .12)^8$
 $y = \$6473.42$

5) The current value of Connie's car is \$5300. The value of her car has been depreciating 22% per year.

- a) What is the percent written as a decimal? .22
- b) Write an exponential equation. $y = 5300(1 - .22)^t$
- c) She hopes to keep the car for 4 more years. What will it be worth then?
 $y = \$1961.80$

6) In 2000, the population of Anytown was 250,000. It has been declining at a rate of 3.2% each year.

- a) What is the percent written as a decimal? .032
- b) Write an exponential equation. $y = 250,000(1 - .032)^t$
- c) What will be its population in 2020?
 $y = 250,000(1 - .032)^{20}$
 $= \$130,450.92$

7) In 2000, Betterville had a population of 175,000 and was growing at a rate of 5.6% each year.

- a) What is the percent written as a decimal? .056
- b) Write an exponential equation. $y = 175,000(1 + .056)^t$
- c) What will be its population in 2020?
 $y = 175,000(1 + .056)^{20}$
 $= \$520,374.99$

8) In 2005, Sally's car was worth \$2300 and has been appreciating 21% each year.

- a) What is the percent written as a decimal? .21
- b) Write an exponential equation. $y = 2300(1 + .21)^t$
- c) What will be its value in 2010? $y = 2300(1 + .21)^5 = 5965.61$

1. \$15,000 is deposited in an account that pays 3.5% annual interest, compounded quarterly. Find the value of the account after 10 years.

$$y = 15000 \left(1 + \frac{.035}{4}\right)^{4(10)}$$

$$= \$21,253.63$$

2. You borrowed \$30,200 for 15 years at an interest rate of 13.6% and it's compounded semi annually. How much in total will you have paid after 15 years?

$$y = 30200 \left(1 + \frac{.136}{2}\right)^{2(15)}$$

$$= 3651.61$$

3. You deposit \$2000 in an account that earns 5% annual interest. Find the balance in the account at the end of 2 years if the account is compounded monthly.

$$y = 2000 \left(1 + \frac{.05}{12}\right)^{12(2)}$$

$$= 2209.88$$