# Unit 8 Day 1 Notes 

## Comparing Linear, Exponential \& Quadratic Functions

To recognize if a function is linear, quadratic (a parabola), or exponential without an equation or graph, look at the differences of the $y$-values between successive integral $x$-values. If the difference is constant, the graph is linear. If the difference is not constant but the second set of differences are constant, the graph is quadratic. If the differences follow a pattern similar to the $y$-values, the graph is exponential. See the examples below for clarity.

## Examples

Based on each table, identify the shape of the graph.

## Example 1

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -7 | -5 | -3 | -1 | 1 | 3 | 5 |



## Example 2

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 9 | 4 | 1 | 0 | 1 | 4 | 9 |



## Example 3

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 | 8 |



Based on the difference in $y$-values, identify the graph as linear, quadratic, exponential, or neither.
1.

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 14 | 10 | 6 | 2 | -2 | -6 | -10 |

3. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 21 | 12 | 5 | 0 | -3 | -4 | -3 |

5. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -14 | -9 | -4 | 1 | 6 | 11 | 16 |

7. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 4 | 8 | 16 | 32 | 64 | 128 | 256 |

9. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 30 | 20 | 12 | 6 | 2 | 0 | 0 |

11. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $1 / 9$ | $1 / 3$ | 1 | 3 | 9 | 27 | 81 |

2. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $1 / 2$ | 1 | 2 | 4 | 8 | 16 | 32 |

4. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -16 | -13 | -10 | -7 | -4 | -1 | 2 |

6. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -18 | -6 | -2 | 0 | 2 | 6 | 18 |

8. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $1 / 27$ | $1 / 9$ | $1 / 3$ | 1 | 3 | 9 | 27 |

10. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 11 | 9 | 7 | 5 | 3 | 1 | -1 |

12. 

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -27 | -9 | -3 | 0 | 3 | 9 | 27 |

Identify the following equations as linear, quadratic or exponential.

| 1. | 2. $y=5+7(x)$ <br> - linear <br> quadratic <br> exponential |
| :---: | :---: |
| 3. $y=(x+3)^{2}-4$ <br> [. linear <br> $\square$ quadratic <br> $\square$ exponential | 4. $y=-2(x)+5$ <br> - linear <br> quadratic <br> exponential |
| 5. $y=-\frac{1}{2}(3)^{x}$ पlinear $\square$ quadratic $\square$ exponential | 6. $y=\frac{1}{3}(x)^{2}-4$ linear $\square$ quadratic $\square$ exponential |

All linear functions can be written as
All exponential functions can be written as $\qquad$ .

All quadratic functions can be written as $\qquad$ .

## Day 1 Homework

Identify the following as Increasing Linear, Decreasing Linear, Positive Quadratic, Negative Quadratic, Exponential Growth, or Exponential Decay.
1.

2.

3.

4.

5.

6. $\qquad$


7.

| $x$ | -2 | -1 | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 100 | 10 | 1 | $\frac{1}{10}$ | $\frac{1}{100}$ |

9. 

| $x$ | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 22 | 17 | 12 | 7 | 2 |

8. $\qquad$

| $x$ | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 1 | 4 | 7 | 10 | 13 |

10. $\qquad$

| $x$ | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $\frac{1}{3}$ | 1 | 3 | 9 | 27 |

11. 

$$
y=\left(\frac{5}{2}\right)^{x}
$$

13. 

$$
y=-2 x-10
$$

15. 

$y=4 x-3$
17.

$$
\mathrm{y}=3 \cdot\left(\frac{1}{4}\right)^{\mathrm{x}}
$$

19. 

$$
y=(x+2)^{2}
$$

21. 

$$
y=2 \cdot 5^{x}
$$

23. 

$$
y=-6 x^{2}-5 x+4
$$

12. $\qquad$

$$
y=\frac{1}{4} \cdot 3^{x}
$$

14. $\qquad$

$$
y=2 x^{2}+5 x-7
$$

16. $\qquad$

$$
\mathrm{y}=\frac{2}{5} \cdot 9^{\mathrm{x}}
$$

18. 

$$
y=2(0.1)^{x}
$$

20. $\qquad$

$$
4 x+y=7
$$

22. 

$$
y=-(x-3)^{2}
$$

24. 

$$
\mathrm{y}=\frac{1}{7} \cdot\left(\frac{3}{8}\right)^{\mathrm{x}}
$$

## Unit 8 Day 2 Notes

For each scenario, draw a sketch of the graph and label the $x$-intercepts and vertex. Describe each point in context.

1. An eagle flies over a canyon. The eagle is 30 feet above the canyon's edge when it drops a stick from its claws. The function $d(t)=-16 t^{2}+30$ gives the height of the stick after $t$ seconds.
2. A sky rocket is shot into the air. Its altitude $h$ in feet after $t$ seconds is given by the function $h(t)=-16 t^{2}+128 t$.
3. The height, in feet, that a certain arrow will reach $t$ seconds after being shot directly upward is given by the formula $h(t)=112 t-16 t^{2}$.
4. Tim launched a rocket from 50 feet above the ground. The height above ground level $h$ of the rocket after $t$ seconds is given by $d(t)=50+45 t-5 t^{2}$.

## Day 2 Practice

2. A rocket carrying fireworks is launched from a hill 80 feet above a lake. The rocket will fall into the lake after exploding at its maximum height. The rocket's height above the surface of the lake is given by the function $h(t)=-16 t^{2}+64 t+80$.
a. What is the height of the rocket after 1.5 seconds?
b. What is the maximum height reached by the rocket?
c. After how many seconds after it is launched will the rocket hit the lake?
3. A rock is thrown from the top of a tall building. The distance, in feet, between the rock and the ground $t$ seconds after it is thrown is given by $\mathrm{d}(\mathrm{t})=-16 t^{2}-4 t+382$. How long after the rock is thrown is it 370 feet from the ground?
4. From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height of the ball $t$ seconds after Susan throws it is given by $h(t)=16 t^{2}+32 t+4$.
a. Find the maximum height reached by the ball and the time this height is reached.
b. When was the ball at the same height as when it was thrown?
5. Marta throws a baseball with an initial upward velocity of 70 feet per second. This equation $h(t)=16 t^{2}+70 t$ models the situation.
a. Ignoring Marta's height, how long after she releases the ball, will it hit the ground?
b. What is the maximum height of the baseball?
6. A volcanic eruption blasts a boulder upward with an initial velocity of 240 feet per second. This is modeled by the equation $h(t)=16 t^{2}+240 t$.
a. How long will it take the boulder to hit the ground?
b. How high was the boulder after 5 seconds?

## Day 4 Practice

1. Which choice is the graph of $y=(4-x)(x+2)$ ?


B


C


D

2. Which expression is equivalent to $(x+2)(3 x-3)$ ?

A $3 x^{2}-6$
B $\quad 3 x^{2}+3 x-6$
C $\quad 3 x^{2}+6 x-6$
D $\quad 3 x^{2}+9 x-6$
3. What is the distance, in units, between the $y$-intercept of $f(x)=x^{2}+7 x-18$ and the $y$-intercept of the linear function that passes through the points shown in the table below?

| $\boldsymbol{x}$ | $\boldsymbol{g ( x )}$ |
| :---: | :---: |
| -5 | 2 |
| 10 | 11 |
| 25 | 20 |
| 60 | 41 |

4. Oscar planted a tomato seed in his garden. Each day he recorded the height of the tomato plant.


During which interval did the tomato plant grow the fastest?
A Day 4 to Day 6
B Day 6 to Day 8
C Day 8 to Day 10
D Day 10 to Day 12
5. Two functions are shown below.

$$
\begin{gathered}
f(x)=3 x^{2}+14 x-5 \\
g(x)=11 x+13
\end{gathered}
$$

Select (click) the points at which the graphs of the two functions intersect.
$(-5,0) \quad(-3,-20) \quad(2,35) \quad(6,79)$
6.

34 The perimeter of the triangle below is $8 x-6$.


Which expression represents the length of $\overline{Q R}$ ?
A $4 x-4$
B $\quad 4 x-6$
C $6 x-4$
D $\quad 6 x-8$
7. What are the solutions to the equation $4 x^{2}-52 x+169=121$ ?

A $\{1,-12\}$
B $\quad\{-1,12\}$
C $\{-1,-12\}$
D $\quad\{1,12\}$
8. David has a rectangle and a right triangle.

- The length of the rectangle is 5 more than its width, $w$.
- The length of the shorter leg of the triangle is equal to the rectangle's width.
- The length of the longer leg of the triangle is twice the length of the rectangle.

Which function, $f(w)$, represents the combined area of the rectangle and the triangle?

A $\quad f(w)=2 w^{2}+10 w$
B $f(w)=3 w^{2}+15 w$
C $\quad f(w)=w^{2}+10 w+25$
D $\quad f(w)=w^{2}+15 w+50$
9. Two functions are shown below.

$$
\begin{gathered}
f(x)=3 x+7 \\
g(x)=2 x+12
\end{gathered}
$$

What is the value of $x$ where the graphs of $f(x)$ and $g(x)$ intersect?
A $\quad-22$
$\begin{array}{ll}B & -5\end{array}$
C 5
D 22
10. A function is shown below.

$$
g(x)=19.60+1.74 x
$$

What is the value of $g(30)$ ?
11. Select (click) each situation that can be modeled with a linear function.

A taxi charges an initial fee of $\$ 2.00$, and $\$ 1.50$ for each additional mile.
The population in a town decreases by $15 \%$ each year.
An airplane flying at an altitude of 33,000 feet descends at a rate 20 feet per minute,
A pizza restaurant charges $\$ 5.50$ per pizza, and $\$ 0.50$ for each additional topping.
A cell doubles in size every 2 hours.
12. What is the distance between the $y$-intercept of the function
$(x)=2 x^{2}-6 x+3$ and the $y$-intercept of the linear function $g$ represented by the table below?

| $\boldsymbol{x}$ | $\boldsymbol{g ( x )}$ |
| :---: | :---: |
| -5 | 15 |
| -2 | 3 |
| 2 | -13 |
| 5 | -25 |

A 2 units
B 3 units
C 8 units
D 9 units
13. A rectangle has a perimeter of 64 .

- Let $x$ equal the width of the rectangle.
- Let $y$ equal the area of the rectangle.

Which equation can be used to find the area of the rectangle?
A $y=x^{2}-64 x$
B $\quad y=-x^{2}+64 x$
C $y=x^{2}-32 x$
D $y=-x^{2}+32 x$

## Unit 8 Day 6 Notes <br> What is a Greatest Common Factor?

Perhaps, the process of factoring by removing the greatest common factor can be best stated as the reverse distributive property. In the distributive property, one is multiplying a certain factor to all of the terms. In factoring by $G C F$, one is dividing all of the terms by the $G C F$.

Consider this expression which utilizes the distributive property: $5 x^{2}\left(4 x^{4}+3\right)$.

Visually, this is the distributive process: $5 x^{2}\left(4 x^{4}+3\right)$.
To simplify using the distributive property, one multiplies $5 x^{2}$ times $4 x^{4}$, and then one multiplies $5 x^{2}$ times 3 .

After simplifying using the distributive property, you get

Factor the greatest common factor: $8 y^{5}-12 y^{3}+4 y$.

Factor the greatest common factor: $14 z^{8}+24 z^{7}-30 z^{3}$.

Factor the greatest common factor: $16 c^{7}-6 c^{3}$.

Factor the greatest common factor: $28 a^{3} b^{2}-36 a^{2}-17 b^{5}$.
Note that the GCF of the coefficients ( $28,-36$, and -17 ) is 1 . Also, note that the terms do not all share any common variables.

Obviously, it makes little sense to write $1\left(28 a^{3} b^{2}-36 a^{2}-17 b^{5}\right)$.

> When one is only factoring out the greatest common factor, and the GCF is 1 , he/she should write that the expression is PRIME .

Factor the greatest common factor out of the polynomial. If the GCF is 1, write PRIME.

1. $8 x^{2}+10 x$
2. $12 y-16$
3. $-15 d^{5}+45 d^{3}$
4. $13 a+20 b$
5. $c^{3}+c^{2}-c$
6. $6 n^{2}-30 n+42$
7. $-7 m^{2}-10 m+17$
8. $18 p^{3}-63 p^{2}-9 p$
9. $18 x^{2}-50 y^{2}$

## Factoring by Grouping (4 terms)

Factor $2 x^{3}-8 x^{2}+5 x-20$.
Group pairs of terms together.
Factor the GCF from each pair.
Write your answer as two factors.

Factor each polynomial by grouping.

1. $2 x^{3}+4 x^{2}+x+2$
2. $2 x^{3}+6 x^{2}+3 x+9$
3. $5 x^{3}-25 x^{2}+2 x-10$
4. $2 x^{3}+12 x^{2}-5 x-30$
5. $7 x^{3}-4 x^{2}+7 x-4$
6. $9 x^{3}-12 x^{2}-18 x+24$

You Try the following. (Notice in \#1-4, they did half the work for you!)

1. $x(a+2)-2(a+2)$
2. $3(x+y)+a(x+y)$
3. $m(x-3)+k(x-3)$
4. $a(y+1)-b(y+1)$
5. $x^{2}+3 x+2 x y+6 y$
6. $y^{2}-5 w y+4 y-20 w$

## Day 6 Homework

For each problem below, factor by finding the GCF.

| 1) $2 a^{4}+8 a$ | 2) $5 x^{3}-10$ |
| :--- | :--- |
| 3) $8 a b^{2}-12 a^{2} b^{3}$ | 4) $10 c^{3} d^{2}-15 c d^{3}$ |
| 5) $15 f-20 g^{2}$ | 6) $3 y^{4}+9 y^{2}-15$ |

Factor each of the following polynomials by grouping
A) $x^{3}-x^{2}+2 x-2$
B) $x^{3}+5 x^{2}-5 x-25$
C) $x^{2}-a x+c x-a c$
D) $5 x^{3}-10 x^{2}+3 x-6$
E) $x^{3}-4 x^{2}+6 x-24$
F) $x^{2}+2 x y+y^{2}-z^{2}$
G) $10 x^{3}+8 x^{2}+15 x y+12 y$
H) $2 x^{3}-10 x^{2}+4 x-20$
I) $3 x^{2}+x y-3 x z-y z$

## Day 9 Homework

Complete each of the following. Part of each answer will be used in the following problem. Write answers in standard form.

Multiply:
$(x+1)(x+8)=$ $\qquad$ The second term's coefficient:

Simplify: $\left(3 x^{2}-x+\square\right)-\left(-4 x^{2}+x-9\right)=$ $\qquad$ The last term: $\qquad$
 The least constant: $\qquad$

Square:
$(\square x+3)^{2}=$ $\qquad$ The second term's coefficient: $\qquad$

Solve for x :

$$
\square x^{2}-84 x-96=0
$$

1. A parabola $\qquad$ has an axis of symmetry.
(a) always
(b) sometimes
(c) never
2. A ball is thrown into the air with an upward velocity of $36 \mathrm{ft} / \mathrm{s}$. Its height $h$ in feet after $t$ seconds is given by the function $h=-16 t^{2}+36 t+9$.
(a) In how many seconds does the ball reach its maximum height?
(b) What is the ball's maximum height?
3. Solve $x^{2}=24-10 x$
4. Solve $z^{2}-6 z-27=0$
5. Factor $12 d^{2}+4 d-1$
6. Factor $r^{2}-49$
$\qquad$

# Cumulative Review 

(Take-Home Quiz Grade!)

## Show all work for credit!

1. Solve $4(x+3)-5=2 x+8$
2. Graph the system $\left\{\begin{array}{l}y \leq 2 x-4 \\ 2 x+y \geq 9\end{array}\right.$
3. For what positive integer value
 will $2^{x}$ first exceed $3 x+2$ ?
4. The expression $-3 m^{2}+15 m$ is the profit for a rock concert based on the ticket price $m$. What is the most the promoters can charge per ticket and still make a profit?
5. Jack's bowling score was 20 less than twice Jill's score. The sum of their scores was 205. What was Jack's score?
6. Find the value for y : $\left\{\begin{array}{l}3 x+y=14 \\ 3 x=2 y-10\end{array}\right.$
7. What are the $x$-intercepts of $f(x)=x^{2}+4 x-5$ ?
8. The Rocket Coaster has 15 cars, some that hold 4 people and some that hold 6 people. There is room for 72 people altogether. How many 4 passenger cars are there?
9. Write the equation of the line perpendicular to $y=\frac{3}{2} x-4$ that passes through the point $(0,5)$.
10. Factor $36 \mathrm{x}^{2}-25$
