Unit 2 Study Guide

Section 1: Exponential Functions

1) The half life of caffeine is 5 hours. A grande Peppermint Mocha has 330 milligrams of caffeine. Let $Q(t)$ denote the amount of caffeine in your system t hours after consuming your grande Peppermint Mocha. For simplicity, assume the entire grande Peppermint Mocha is consumed instantly.
   a. How many milligrams of caffeine will be in your system after 5 hours? After 10 hours? After 15 hours? 5 hours = 330 mg 15 hours = 82.5
   10 hours = 165 mg
   b. $Q(t) = Q_0 \frac{1}{2}^{t/k}$. Find Q_0 and k. $Q(t) = 330\left(\frac{1}{2}\right)^{t/5}$
   Q_0 is the original amount of caffeine
   k is the half life of caffeine
   c. How many milligrams of caffeine will be in your system after 2 hours?
   $Q(t) = 330\left(\frac{1}{2}\right)^{2/5}$
   250.093 mg

2) A bacteria culture triples in size every 7 hours. Three hours from now, the culture has 8,000 bacteria. If $Q(t)$ denotes the number of bacteria, then $Q(t) = Q_0 e^{kt}$ for some number Q_0 and for some number k.
   a. Determine Q_0 and k.
   b. How many bacteria are there at time $t = 0$?
   c. How many bacteria are there after ten hours?

3) The world population in 2000 was approximately 6.08 billion. The annual rate of increase was about 1.26%
   a. Find the growth factor for the world population
   $(1 + .0126) = 1.0126$
   b. Suppose the rate of increase continues to be 1.26%. Write a function to model the world population.
   $y = 6.08(1 + .0126)^x$
   c. Let x be the number of years past the year 2000. Find the world population in 2010.
   $y = 6.08(1 + .0126)^{10}$
   6.89 billion people in 2010

*look over transformations for exponential functions

$\log_b y = x \implies b^x = y$

Section 2: Convert each equation to either logarithm equations or exponential functions

4) $\log_2(x) = 4$

$2^4 = x$

5) $\frac{1}{2} = \log_x 49$

$x^{\frac{1}{2}} = 49$

6) $y = \log_7 9$

$7^y = 9$

7) $6^x = 53$

$\log_6 53 = x$

8) $x^5 = 73$

$\log_x 73 = 5$

9) $98 = 7^y$

$\log_7 98 = y$

***look over definitions for common logarithm and natural logarithm**

Section 3: Expand or Condense Logarithms

10) $3\log_9 2 - 2\log_9 5$

$\log_9 \frac{2^3}{5^2}$

11) $\log_6 x + \log_6 y + 6\log_6 z$

$\log_6(xyz^6)$

12) $2\log_5 x + 12\log_5 y$

$\log_5(x^2y^{12})$

13) $\log_3 12 + \log_3 7 + 4\log_3 5$

$\log_3(12 \cdot 7 \cdot 5^4)$

14) $\log_7 \frac{x^4}{y^2}$

$4\log_7 x - 2\log_7 y$

15) $\log_3(z\sqrt[3]{x \cdot y})$

$\log_3 z + \frac{1}{3}\log_3 x + \frac{1}{3}\log_3 y$

$\boxed{\text{or}}$

$\log_3 z + \frac{\log_3 x}{3} + \frac{\log_3 y}{3}$

16) $\log_6(uv^3)^2$

$2\log_6 u + 6\log v$ $\boxed{\text{or}}$

$2(\log_6 u + 3\log v)$

17) $\log_4(12 \cdot 7^2)^4$

$4\log_4 12 + 8\log_4 7$

Section 4: Solving Exponential and Logarithmic Equations

18) $3^{2n-2} = 9$

$3^{2n-2} = 3^2$

$2n - 2 = 2$

$+2 \quad +2$

$\frac{2n}{2} = \frac{4}{2}$

$\boxed{n = 2}$

19) $625^{3x} = 125^{x+1}$

$5^{4(3x)} = 5^{3(x+1)}$

$4(3x) = 3(x+1)$

$12x = 3x + 3$

$-3x \quad -3x$

$\frac{9x}{9} = \frac{3}{9}$

$\boxed{x = \frac{1}{3}}$

20) $e^{b-3} = 6$

$\ln 6 = b - 3$
$+3 \quad +3$

$\ln 6 + 3 = b$

$\boxed{b = 4.792}$

21) $15^{x-8} + 5 = 62$
$-5 \quad -5$

$15^{x-8} = 57$

$\log_{15} 57 = x - 8$
$+8 \quad +8$

$\log_{15} 57 + 8 = x$

$\boxed{x = 9.493}$

22) $-6 + \log 6v = -7$
$+6 \quad +6$

$\log_{10} 6v = -1$

$\frac{10^{-1}}{6} = \frac{6v}{6}$

$\boxed{v = \frac{1}{60}}$

23) $-5 \log_8 3m = 5$
$\div -5 \quad \div -5$

$\log_8 3m = -1$

$\frac{8^{-1}}{3} = \frac{3m}{3}$

$\boxed{m = \frac{1}{24}}$

24) $\frac{9 \log_7 (k-3)}{9} = \frac{0}{9}$

$\log_7 (k-3) = 0$

$7^0 = k - 3$
$+3 \quad +3$

$\boxed{k = 4}$

25) $\log_4(x+6) + \log_4 x = 2$

$\log_4 (x + 6(x)) = 2$

$\log_4 (x^2 + 6x) = 2$

$4^2 = x^2 + 6x$

$16 = x^2 + 6x$
$-16 \quad -16$

$0 = x^2 + 6x - 16$

$0 = (x-2)(x+8)$

$\boxed{x = 2, -8}$

Section 5: Compound Interest

26) If, at the end of two years, a savings account has a balance of $1172.60, and the interest rate is compounded monthly at 3.2%, then what is the original amount deposited two years ago?

$$y = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

$$\frac{1172.60}{\left(1 + \frac{.032}{12}\right)^{12 \cdot 2}} = \frac{P\left(1 + \frac{.032}{12}\right)^{12 \cdot 2}}{\left(1 + \frac{.032}{12}\right)^{12 \cdot 2}}$$

$P = 1099.99$

$\boxed{\text{or}}$

1100.00

27) A teenager saved small dollar amounts throughout the school year and now has $712.00. They can choose from two bank offers. The first is 5.3% compounded continuously for six years. The second is compounded quarterly for five years at 6.0%. Which account will yield the most money? What is the dollar amount difference between the accounts at the end of their terms?

$y = Pe^{rt}$

$y = 712 e^{.053(6)}$

$y = 978.56$

$y = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

$y = 712\left(1 + \frac{.06}{4}\right)^{4 \cdot 5}$

$y = 958.96$

The first bank offer yield the most money.
The difference is $19.60.

[redacted]

Section 6: Simplify the following

28) $(x^{-2}x^{-3})^4$

$x^{-8}x^{-12} = \frac{1}{x^8x^{12}}$

$= \boxed{\frac{1}{x^{20}}}$

29) $\frac{2x^2y^4 \cdot 4x^2y^4 \cdot 3x}{3x^{-3}y^2}$

$\frac{24x^5y^8}{3x^{-3}y^2} = \boxed{8x^8y^6}$

30) $\frac{3x^2y^2}{2x^{-1} \cdot 4yx^2}$

$\frac{3x^2y^2}{8xy} = \boxed{\frac{3}{8}xy}$

31) $4a^3b^2 \cdot 3a^{-4}b^{-3}$

$12a^{-1}b^{-1} = \boxed{\frac{12}{ab}}$

32) $2x^3y^{-3} \cdot 2x^{-1}y^3$

$4x^2y^0 = \boxed{4x^2}$

33) $(2x^2)^{-4}$

$2^{-4}x^{-8} = \frac{1}{2^4x^8} = \boxed{\frac{1}{16x^8}}$

34) $\frac{3x^3y^{-1}z^{-1}}{x^{-4}y^0z^0} = 3x^7y^{-1}z^{-1}$

$= \boxed{\frac{3x^7}{yz}}$

34) $2y^2 \cdot 3x$

$\boxed{6xy^2}$

Section 7: Newton's Law of Cooling

Equation: $T = (T_0 - T_R)e^{-rt} + T_R$

T = temperature of the substance

T_R = room temperature

t is time in minutes

T_0 = initial temperature

r = constant cooling rate of the substance

35) A pizza is taken from a 425°F oven and placed on the counter to cool. If the temperature in the kitchen is 75°F, and the cooling rate for this type of pizza is $k = 0.35$

a. What is the temperature (to the nearest degree) of the pizza 2 minutes later?

$T = (425 - 75)e^{-0.35(2)} + 75$

$= \boxed{248.805°F}$

b. To the nearest minute, how long until the pizza has cooled to a temperate below 90°F?

$90 = (425 - 75)e^{-0.35t} + 75$

$-75 \qquad -75$

$\frac{15}{350} = \frac{350e^{-0.35t}}{350} \Longrightarrow \frac{3}{70} = e^{-0.35t}$

$\frac{\ln\frac{3}{70}}{-0.35} = \frac{-0.35t}{-0.35}$

less than

$\boxed{t = 9 \text{ minutes}}$

c. If Matt and Tyler like to eat their pizza at a temperature of about 110°F, how many minutes should they wait to "dig in"?

$110 = (425 - 75)e^{-0.35t} + 75$

$-75 \qquad -75$

$\frac{35}{350} = \frac{350e^{-0.35t}}{350} \Longrightarrow \frac{1}{10} = e^{-0.35t}$

$\frac{\ln\frac{1}{10}}{-0.35} = \frac{-0.35t}{-0.35}$

$\boxed{t = 6.58 \text{ minutes}}$