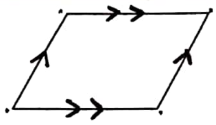
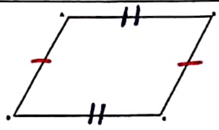
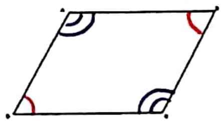
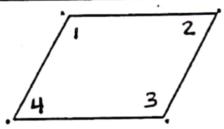
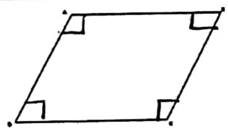
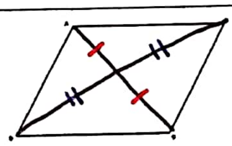
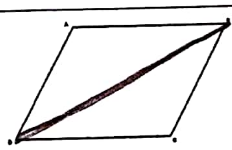
Honors Math 3

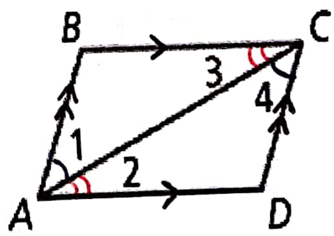
5.5 Parallelograms

Unit 5

EQ: How can we prove a figure to be a parallelogram and solve for variables in a parallelogram?

Properties of Parallelograms		
Sides	A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.	
	If a quadrilateral is a parallelogram, the 2 pairs of opposite sides are congruent.	
Angles	If a quadrilateral is a parallelogram, the 2 pairs of opposite angles are congruent.	
	If a quadrilateral is a parallelogram, the consecutive angles are supplementary. $\angle 1 + \angle 2 = 180$ $\angle 2 + \angle 3 = 180$ $\angle 3 + \angle 4 = 180$ $\angle 4 + \angle 1 = 180$	
	If a quadrilateral is a parallelogram and one angle is a right angle, then all angles are right angles.	
Diagonals	If a quadrilateral is a parallelogram, the diagonals bisect each other.	
	If a quadrilateral is a parallelogram, the diagonals form two congruent triangles.	

Example 1: Given: ▱ABCD is a parallelogram.
Prove: AB ≅ CD and BC ≅ DA.



Statement	Reason
1. ABCD is a parallelogram	1. Given
2. $\overline{AB} \parallel \overline{CD}$ $\overline{BC} \parallel \overline{AD}$	2. Definition of a parallelogram
3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 2$	3. Alt. Int. $\angle$s $\cong$
4. AC ≅ AC	4. Reflexive Property
5. ΔABC ≅ ΔCDA	5. ASA $\triangle \cong$
$\overline{AB} \cong \overline{CD}$ $\overline{BC} \cong \overline{DA}$	6. CPCTC

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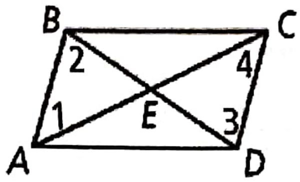
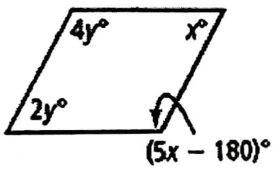
Example 2: Given: ▱ABCD is a parallelogram.
Prove: AC and BD bisect each other at E.



Statement	Reason
1. ABCD is a parallelogram	1. Given
2. AB ∥ DC	2. Def. of ▱
3. ∠1 ≅ ∠4, ∠2 ≅ ∠3	3. Alt. int. ∠s ≅
4. AB = DC	4. Opp. sides of ▱ ≅
5. △ABE ≅ △CDE	5. ASA
6. AE ≅ CE, BE ≅ DE	6. CPCTC (corresponding parts of congruent triangles are congruent)
7. E is the midpoint of $\overline{AC}$ & $\overline{BD}$	7. Definition of midpoint
8. $\overline{AC}$ & $\overline{BD}$ bisect each other @ E.	8. Definition of bisector

Example 3: For what values of x and y must each figure be a parallelogram?

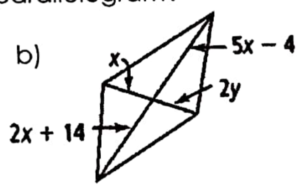
a)



$4y + 2y = 180$
$6y = 180$
$y = 30$

$2(30) = x$
$60 = x$

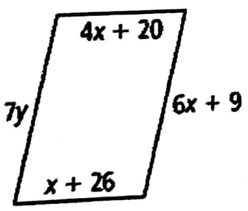
b)



$5x - 4 = 2x + 14$
$3x = 18$
$x = 6$

$6 = 2y$
$y = 3$

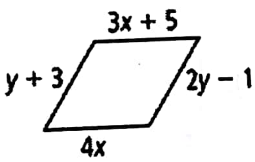
c)



$4x + 20 = x + 26$
$3x = 6$
$x = 2$

$7y = 6(2) + 9$
$7y = 21$
$y = 3$

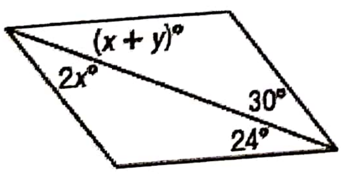
d)



$4x = 3x + 5$
$x = 5$

$2y - 1 = y + 3$
$y = 4$

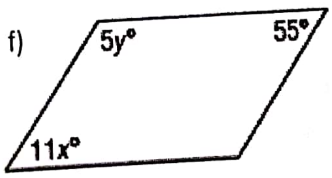
e)



$2x = 30$
$x = 15$

$15 + y = 24$
$y = 9$

f)

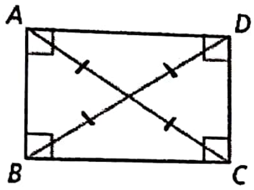
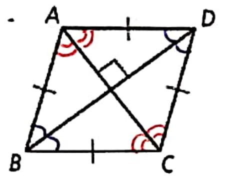
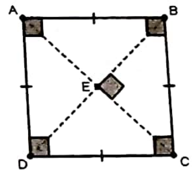


$11x = 55$
$x = 5$

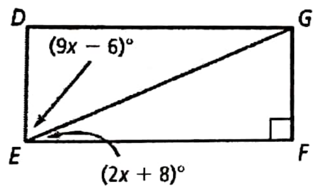
$55 + 5y = 180$
$5y = 125$
$y = 25$

5.5 Quadrilaterals

EQ: How can we use the properties of quadrilaterals to solve for unknowns?

Rectangle	Rhombus	Square
A **rectangle** is a parallelogram with four right angles.	A **rhombus** is a parallelogram with four congruent sides.	A **square** is a parallelogram with four congruent sides and four right angles.
A **rectangle** has all the properties of a parallelogram PLUS: • 4 right angles • Diagonals are congruent	A **rhombus** has all the properties of a parallelogram PLUS: • 4 congruent sides • Diagonals bisect angles • Diagonals are perpendicular	A **square** has all the properties of a parallelogram PLUS: • All the properties of a rectangle • All the properties of a rhombus
		

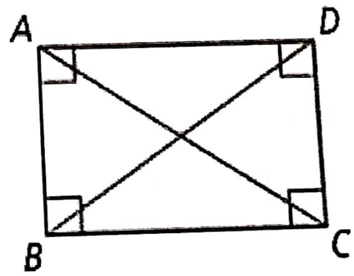
Example 1: Solve for x and the measure of each angle if ▭DGFE is a rectangle.



$9x - 6 + 2x + 8 = 90$
$11x + 2 = 90$
$11x = 88$
$x = 8$

$m\angle DEG = 66°$
$m\angle FEG = 24°$

Example 2: ▭ABCD is a rectangle whose diagonals intersect at point E.



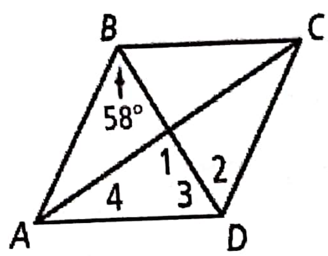
a) If AE = 36 and CE = 2x – 4, find x.

$2x - 4 = 36$
$2x = 40$
$x = 20$

b) If BE = 6y + 2 and CE = 4y + 6, find y.

$6y + 2 = 4y + 6$
$2y = 4$
$y = 2$

Example 3: Using the diagram to the right to answer the following if ▭ABCD is a rhombus.



a) Find the m∠1.

90°

b) Find the m∠2.

58°

c) Find the m∠3.

58°

* diagonals bisects angles

d) Find the m∠4.

$180 - 90 - 58 = 32°$

Key

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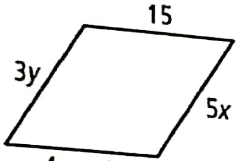
Example 4: Solve for each variable if the following are rhombi.

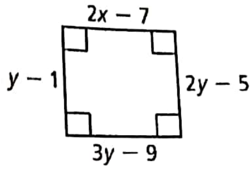
a)



$4x + 3 = 15$
$4x = 12$
$x = 3$

$3y = 5(3)$
$3y = 15$
$y = 5$

b)



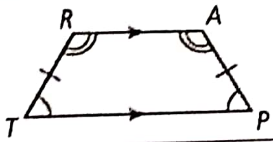
$2y - 5 = y - 1$
$y = 4$

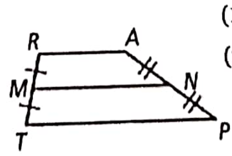
$2x - 7 = 3(4) - 9$
$2x - 7 = 3$
$2x = 10$
$x = 5$

Trapezoid

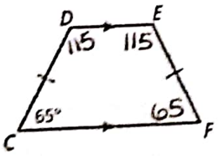
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides, called bases, and two nonparallel sides, called *legs.*

Isosceles Trapezoids	Trapezoid Midsegment
An **isosceles trapezoid** is a trapezoid with congruent legs.	The **median** (also called the midsegment) of a trapezoid is a segment that connects the midpoint of one leg to the midpoint of the other leg.
A trapezoid is isosceles if there is only: • One set of parallel sides • Base angles are congruent • Legs are congruent • Diagonals are congruent • Opposite angles are supplementary $\angle T \cong \angle P, \angle R \cong \angle A$	**Theorem:** If a quadrilateral is a trapezoid, then a) the midsegment is parallel to the bases and b) the length of the midsegment is half the sum of the lengths of the bases (1) $\overline{MN} \parallel \overline{TP}, \overline{MN} \parallel \overline{RA}$, and (2) $MN = \frac{1}{2}(TP + RA)$



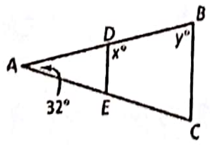


Example 5: CDEP is an isosceles trapezoid and m<C = 65. What are m<D, m<E, and m<F?



$$\begin{array}{r} 180 \\ -\ 65 \\ \hline 115 \end{array}$$

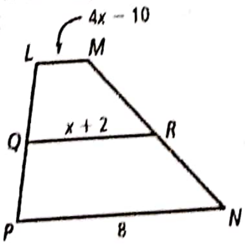
Example 6: What are the values of x and y in the isosceles triangle below if DE || DC?



$32 + 2y = 180$
$2y = 148$
$y = 74$

$180 - 74 = x$
$x = 106$

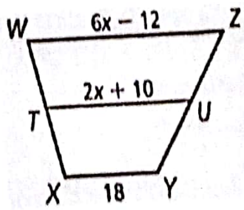
Example 7: QR is the midsegment of trapezoid LMNP. What is x and the length of



$\frac{4x - 10 + 8}{2} = x + 2$
$4x - 2 = 2x + 4$
$2x = 6$
$x = 3$

$= 2$

You Try! TU is the midsegment of trapezoid WXYZ. What is x and the length of TU?



$\frac{6x - 12 + 18}{2} = 2x + 10$
$6x + 6 = 4x + 20$
$2x = 14$
$x = 7$

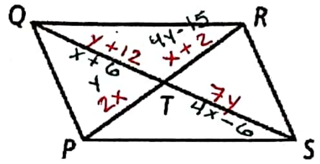
$TU = 24$

...allelograms & Quadrilaterals

...ors Math 3

Name: ____________________

1. Use the diagram below to solve for x and y if the figure is a parallelogram.



a) PT = 2x, QT = y + 12, TR = x + 2, TS = 7y

$2x = x + 2$

$\boxed{x = 2}$

$7y = y + 12$

$6y = 12$

$\boxed{y = 2}$

b) PT = y, TR = 4y - 15, QT = x + 6, TS = 4x - 6

$y = 4y - 15$

$-3y = -15$

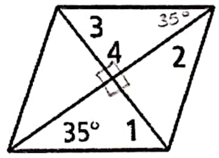
$\boxed{y = 5}$

$x + 6 = 4x - 6$

$-3x = -12$

$\boxed{x = 4}$

2. Find the measure of each angle if the figure is a rhombus.



a) Find the m∠1.

55°

b) Find the m∠2.

35°

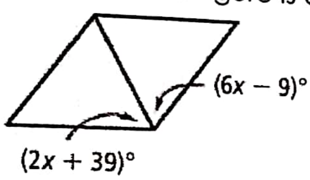
c) Find the m∠3.

55°

d) Find the m∠4.

90°

3. Solve for x if the figure is a rhombus.

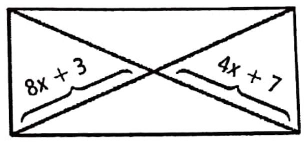


$6x - 9 = 2x + 39$

$4x = 48$

$\boxed{x = 12}$

4. Solve for x if the figure is a rectangle.

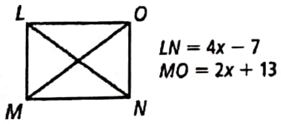


$8x + 3 = 4x + 7$

$4x = 4$

$\boxed{x = 1}$

5. What is the length of LN if the figure is a rectangle?



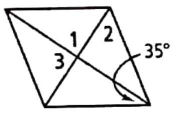
$4x - 7 = 2x + 13$

$2x = 20$

$x = 10$

$\overline{LN} = 33$

6. Solve for the missing angle measures if the figure is a rhombus.



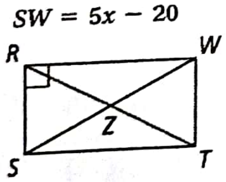
∠1 = 90°

∠3 = 90°

∠2 = 55°

7. What is the length of SW?

RZ = 2x + 5,
SW = 5x − 20



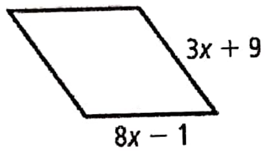
$2(2x + 5) = 5x - 20$

$4x + 10 = 5x - 20$

$30 = x$

$\overline{SW} = 130$

8. Solve for x if the figure is a rhombus.



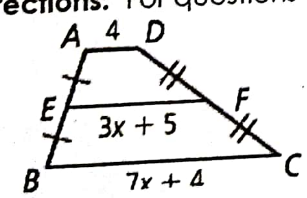
$8x - 1 = 3x + 9$

$5x = 10$

$\boxed{x = 2}$

Directions: For questions #9-10, find x and the length of EF.

9.



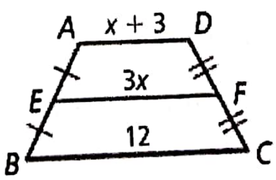
$\frac{7x + 4 + 4}{2} = 3x + 5$

$7x + 8 = 6x + 10$

$x = 2$

$\overline{EF} = 11$

10.



$\frac{x + 3 + 12}{2} = 3x$

$x + 15 = 6x$

$15 = 5x$

$x = 3$

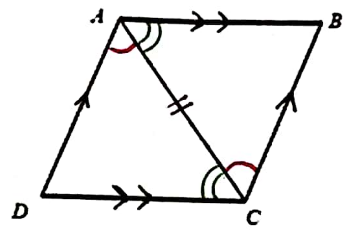
$\overline{EF} = 9$

Proof #1:

Given: ▱$ABCD$

Prove: $\Delta DAC \cong \Delta BCA$

(At most 6 steps! You may not need all 6!!!)

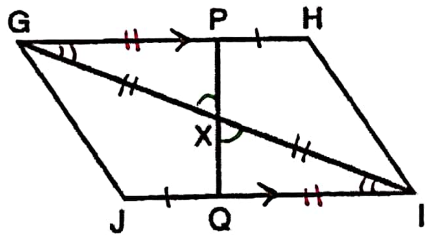


Statements	Reasons
1 ▱ ABCD	1 Given
2 $\overline{AB} \parallel \overline{DC}$ $\overline{AD} \parallel \overline{BC}$	2 Def of ▱
3 $\overline{AC} \cong \overline{AC}$	3 Reflexive Prop.
4 $\angle DAC \cong \angle BCA$	4 Alt. Int. ∠s ≅
5 $\angle ACD \cong \angle CAB$	5 Alt. Int. ∠s ≅
6 $\Delta DAC \cong \Delta BCA$	6 ASA Δ ≅

Proof #2:

Given: ▱GHIJ
$\overline{HP} \cong \overline{JQ}$

Prove: $\overline{PX} \cong \overline{QX}$



Statements	Reasons
1 ▱ GHIJ $\overline{HP} \cong \overline{JQ}$	1 Given
2 $\overline{GH} \parallel \overline{JI}$	2 Def. of ▱
3 $\angle PGX \cong \angle XIQ$	3 Alt. Int. ∠s ≅
4 $\angle PXG \cong \angle QXI$	4 Vert. ∠s ≅
5 $\overline{GH} \cong \overline{JI}$	5 opp. sides of ▱ ≅
6 $\overline{QI} = \overline{JI} - \overline{JQ}$ $\overline{GP} = \overline{GH} - \overline{PH}$	6 seg. add. postulate
7 $\overline{QI} = \overline{GP}$	7 substitution
8 $\Delta GXP \cong \Delta IXQ$	8 AAS ≅
9 $\overline{PX} \cong \overline{QX}$	9 CPCTC (corresponding parts of congruent triangles are congruent)