Day 4 Outline

Warm-up
  1) Show Patterns Video: https://www.youcubed.org/wim-day-4/   (You must be logged in to YouCubed to get video)
  2) Pascal Triangles
  3) Kid President Pep Talk (https://www.youtube.com/watch?v=RwlhUcSGqgs)
HW
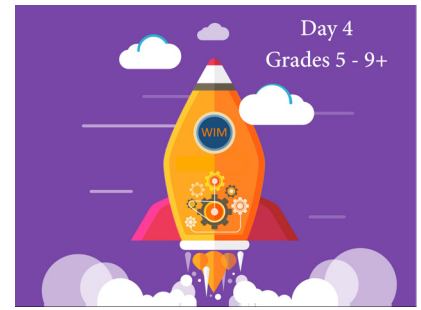
Day 4: Pascal's Triangle

Text by Professor Jo Boaler

Introduction

Many school children, when asked to describe math, will say it is all about rules and procedures. But most mathematicians will tell you that math is about the study of patterns. Keith Devlin, Stanford Mathematician, wrote a book called: 'The Science of Patterns' in which he talks about mathematics being, at its heart, about pattern seeking. In this activity we invite students to be mathematicians and to find and study patterns in the world's most famous triangle that has fascinated people for centuries.

Video

The message of the video is that math is about the study of patterns. Students learn about Fibonacci's sequence and where it exists in nature.

Activity: Exploring Pascal's Triangle

When I trialed this activity I started the lesson by saying that we were going to explore a really famous triangle that is full of patterns, some of which may not even be discovered yet. I told students that the triangle is often named Pascal's Triangle, after Blaise Pascal, who was a French mathematician from the 1600's, but we know the triangle was discovered and used much earlier in India, Iran, China, Germany, Greece and Italy. When I told students this many students, particularly those with ancestry in those countries, seemed excited.

I then gave all the students our copy of Pascal's triangle with some missing numbers and asked students to work in pairs to find the missing numbers. We chose pairs over groups as finding the numbers is not a very open ended task and we did not want students to feel left out, which can happen in a group of 4. After the students had found the missing numbers we gave them the Pascal handout we have created. The handout asks students to first find the sum of the numbers in the rows and then to shade the odd numbers to think about patterns, then we ask students to explore and find their own patterns. We did trial a version of this when we only asked students the more open question of exploring and finding patterns, but we found that was not engaging enough for some of the students. When we trialed this version it resulted in very high levels of engagement.

In the handout we show students what triangular numbers are and ask them to find some triangular numbers. You may prefer, as the teacher, to teach students about triangular numbers if students don't know them and not have this in the worksheet. It is really nice for the students who don't know these numbers to see them, work some more out, and then to see that they are represented in Pascal's triangle:

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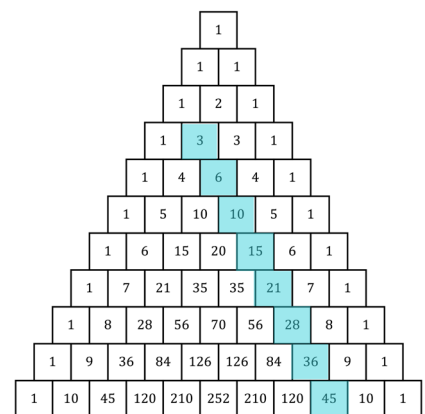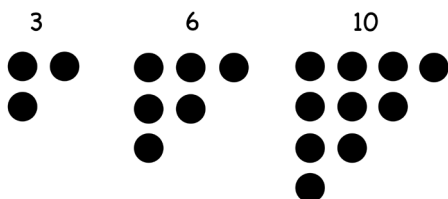
Day 4: Pascal's Triangle

In our trial students worked for some time finding their own patterns and really enjoyed discovering patterns.

A good ending to the class is students sharing the different patterns they have found. We found that students were fascinated by each others patterns and many students wanted to take their triangle home to show them to parents and to study them further.

As extensions to this task we are providing some investigations that produce Pascal's numbers in their solutions. This can be very exciting for students. For example, when students work out how many trains can be made in rod trains, they will see a row of Pascal's triangle. This is a nice follow on activity for the Pascal investigation and could be worked on later in the school year.

Activity	Time	Description/Prompt	Materials
Day 4 Video: Patterns	5 min	Video https://www.youcubed.org/wim-day-4/	
Pascal's Triangle	30 min	1. Introduce Pascal's Triangle 2. Find the missing numbers on the Pascal's Triangle handout while working in pairs (page 4) 3. Investigate the 4 questions on the Pascal handout (page 3)	• Paper, pencil/pen • Colored pencils/markers • Pascal's Triangle Handout, page 4 • Pascal's Investigation Handout, page 3
Group Presentations	10 min	Ask students to share any patterns or other interesting observations	
Closing	5 min	Review the key concepts: Patterns are everywhere and they are very important in math. Patterns help us to connect numbers and visuals which is really good for learning. Remember the brain crossing from day 2?	

Extensions:
• Rod Trains, page 5. This is really good to do with Cuisenaire Rods if you have them!
• Lattice Task, page 6
• Pascal's Triangle with empty rows, page 7
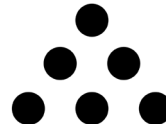
Day 4: Pascal's Triangle

In pairs investigate these patterns.

1. Find the sum of each row in Pascal's Triangle. Is there a pattern?

2. Shade all of the odd numbers in Pascal's Triangle. Is there a pattern?

3. Triangular numbers are numbers that can be drawn as a triangle.

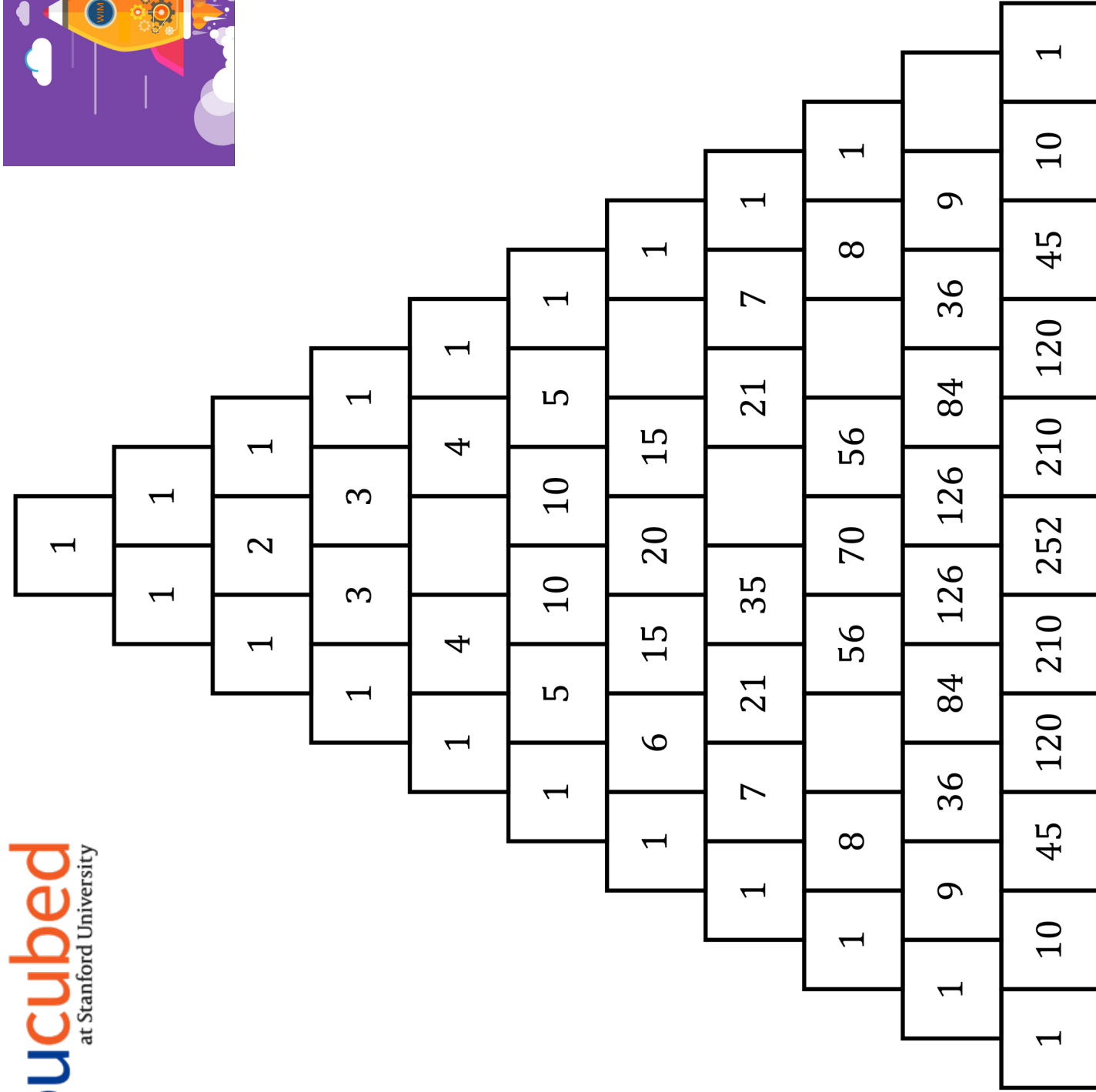> For example, 3 is a triangular number and can be drawn like this.

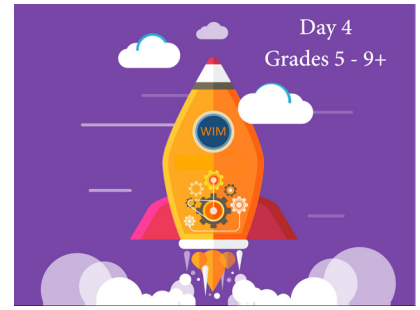> 6 is a triangular number and can be drawn like this.

> Find and represent the next two triangular numbers.

> Can you find triangular numbers in Pascal's Triangle?

4. Find one more pattern in Pascal's Triangle and be prepared to share your findings with the class.

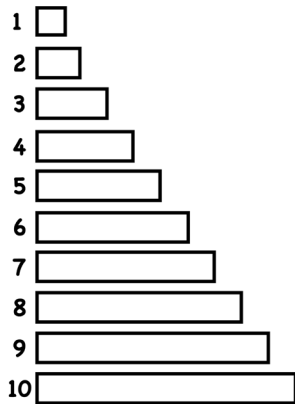|										
1										
1	1									
1	2	1								
1	3	3	1							
1	4		4	1						
1	5	10	10	5	1					
1	6	15	20	15		1				
1	7	21	35		21	7	1			
1	8		56	70	56		8	1		
1	9	36	84	126	126	84	36	9	1	
1	10	45	120	210	252	210	120	45	10	1

Day 4: Pascal's Triangle

Extension Activity: Rod Trains
Optional manipulative, Cuisenaire Rods

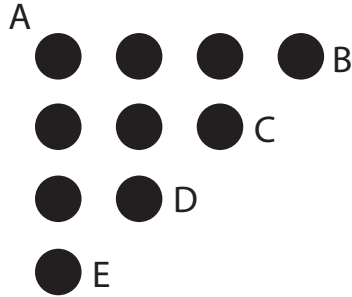Imagine you have rods of unit lengths as in the following diagram.

Find out how many different rod trains can be made from any length of rod. For example, you can make these 4 trains for the 3 rod.
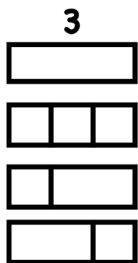
Day 4: Pascal's Triangle

Extension Activity: Lattice

You are at point A. As you move around the grid, you are only allowed to take steps to the right or down. How many ways are there to get to the following points:

B?
C?
D?
E?

How did you find these answers? Can you justify why they are correct? What would happen if you were allowed to move any direction? Would your answers change?

Now suppose we add another diagonal row of dots to our lattice:
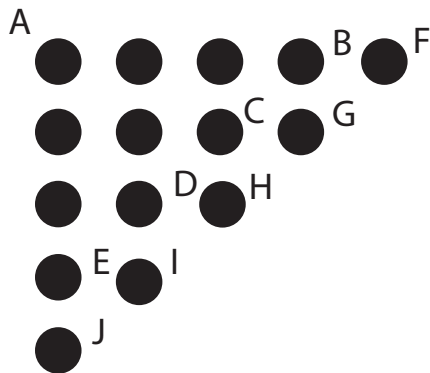
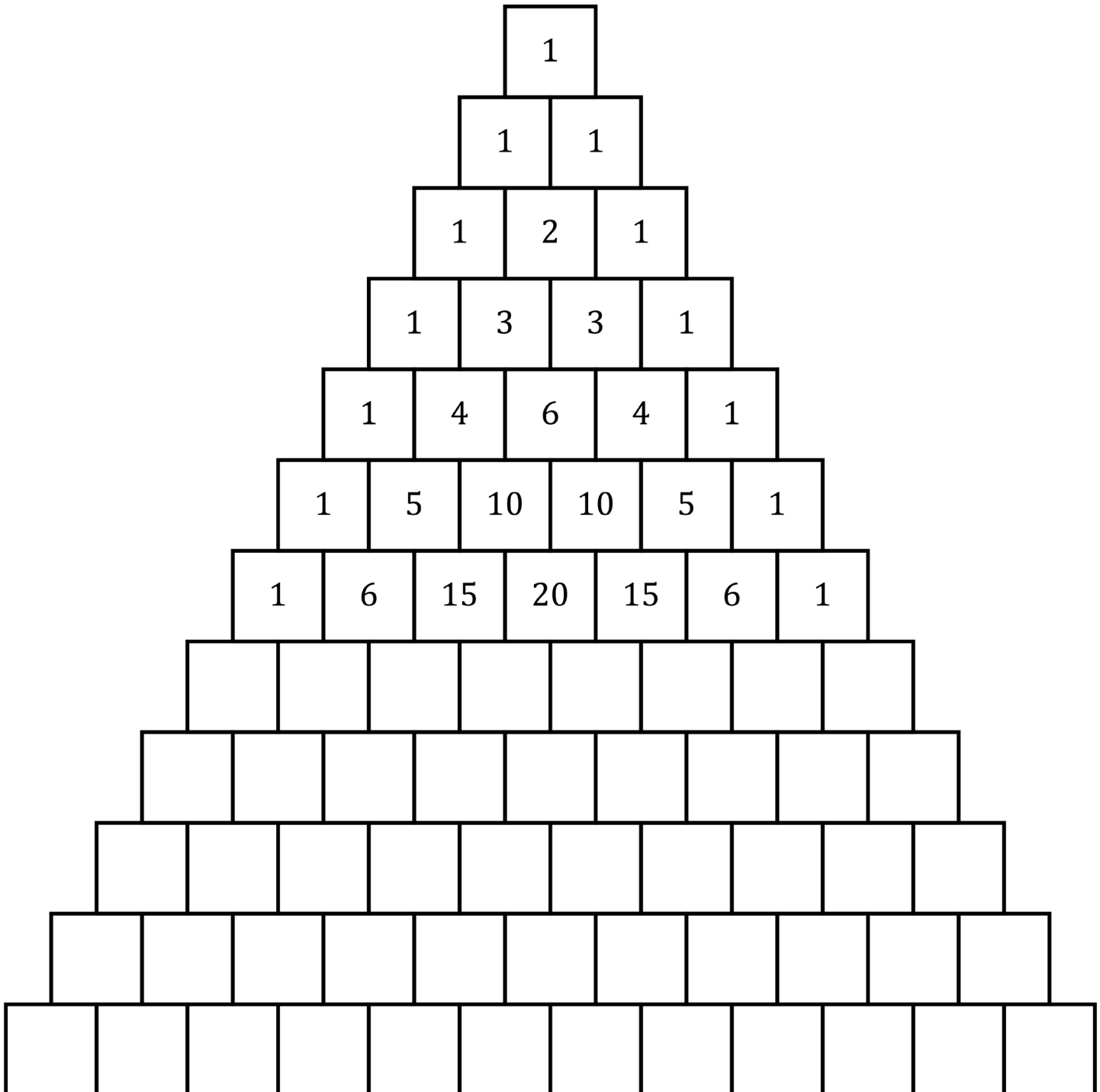How many ways are there to get to all of the new points:

F?
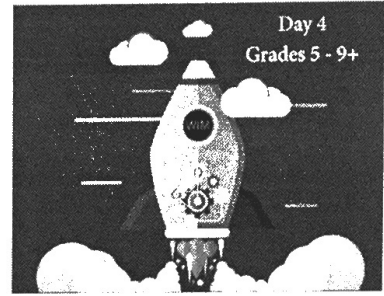G?
H?
I?
J?

How did you find these answers?

Day 4: Pascal's Triangle

Day 4
Grades 5 - 9+

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
1   6  15  20  15   6   1
```

7

Day 4: Pascal's Triangle

In pairs investigate these patterns.

1. Find the sum of each row in Pascal's Triangle. Is there a pattern?

Each sum doubles

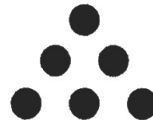2. Shade all of the odd numbers in Pascal's Triangle. Is there a pattern?

yes, each set of 4 ones on the outside diagonal makes up a triangle with the center block missing

3. Triangular numbers are numbers that can be drawn as a triangle.

For example, 3 is a triangular number and can be drawn like this.

6 is a triangular number and can be drawn like this.

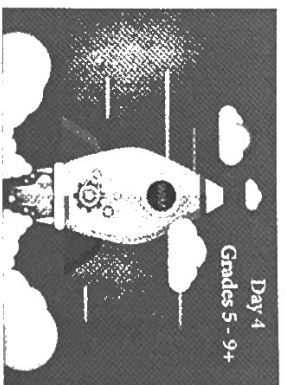Find and represent the next two triangular numbers.

10   15

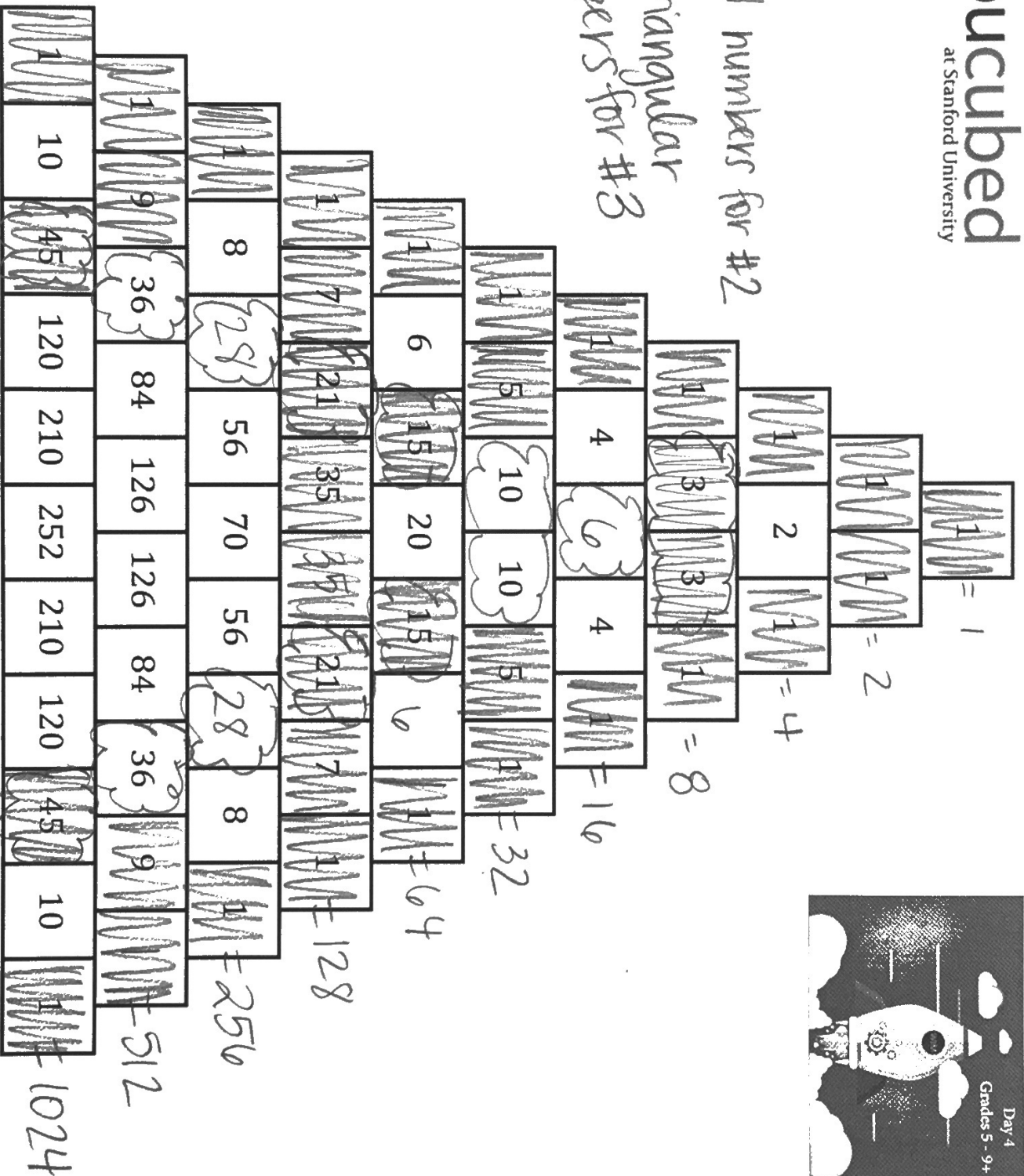Can you find triangular numbers in Pascal's Triangle?
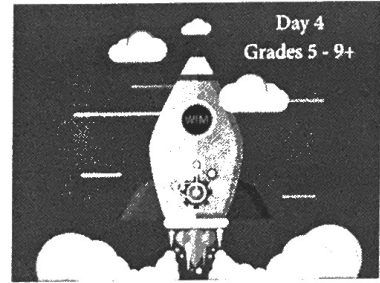
yes, 3, 6, 10, 15, 21, 28, 36, 45

4. Find one more pattern in Pascal's Triangle and be prepared to share your findings with the class.

Ex] The next row is made up of sum's of two numbers above the block in the previous row

4

is odd numbers for #2

is triangular numbers for #3

Pascal's Triangle:

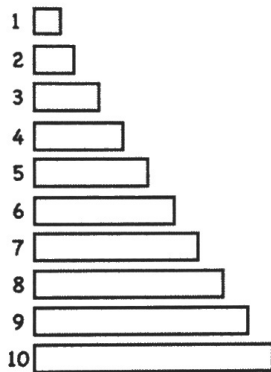|																					
1																					= 1
1		1																			= 2
1		2		1																	= 4
1		3		3		1															= 8
1		4		6		4		1													= 16
1		5		10		10		5		1											= 32
1		6		15		20		15		6		1									= 64
1		7		21		35		35		21		7		1							= 128
1		8		28		56		70		56		28		8		1					= 256
1		9		36		84		126		126		84		36		9		1			= 512
1		10		45		120		210		252		210		120		45		10		1	= 1024

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Day 4: Pascal's Triangle

Extension Activity: Rod Trains
Optional manipulative, Cuisenaire Rods

Imagine you have rods of unit lengths as in the following diagram.

1
2
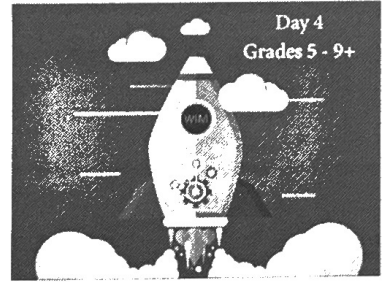3
4
5
6
7
8
9
10

Think of all representations!

Find out how many different rod trains can be made from any length of rod. For example, you can make these 4 trains for the 3 rod.

3

④

⑤
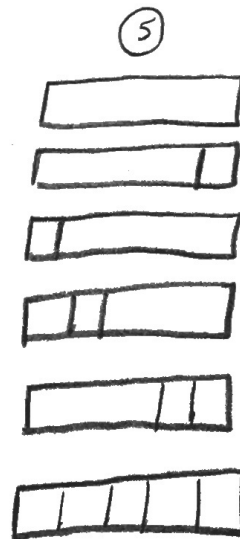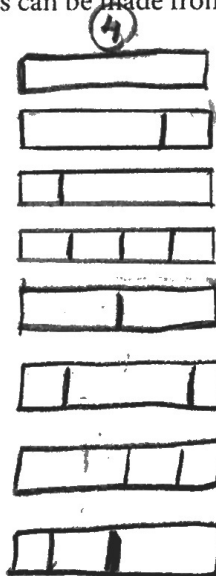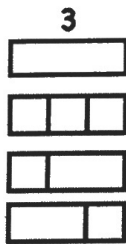
etc

4
3 + 1
1 + 3
1 + 1 + 1 + 1
2 + 2
1 + 2 + 1 ⁵
2 + 1 + 1
1 + 1 + 2

etc.
•
•
•

*see
visual
in key
folder*

Day 4: Pascal's Triangle
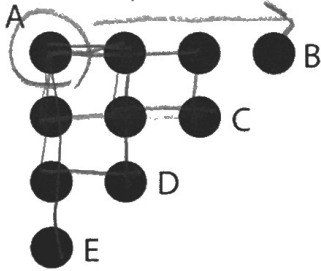
Extension Activity: Lattice

You are at point A. As you move around the grid, you are only allowed to take steps to the right or down. How many ways are there to get to the following points:

B? *1*
C? *3*
D? *2*
E? *1*

How did you find these answers? Can you justify why they are correct? What would happen if you were allowed to move any direction? Would your answers change? *Ask Students*

Now suppose we add another diagonal row of dots to our lattice:
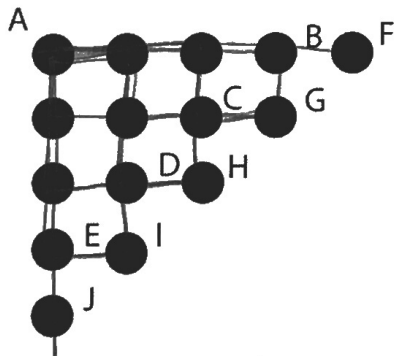
How many ways are there to get to all of the new points:

F? *1*
G? *4*
H? *3*
I? *2*
J? *1*

How did you find these answers?

Ask students 6

Day 4: Pascal's Triangle

					1						
				1		1					
			1		2		1				
		1		3		3		1			
	1		4		6		4		1		
1		5		10		10		5		1	
1	6	15	20	15	6	1					
1	7	21	35	35	21	7	1				
1	8	28	56	70	56	28	8	1			
1	9	36	84	126	126	84	36	9	1		
1	10	45	120	210	252	210	120	45	10	1	
1	11	55	165	330	462	462	330	165	55	11	1

	1 rod	2 rods		
2	1	1		

	1 rod	2 rods	3 rods	
4	1	2	1	

	1 rod	2 rods	3 rods	4 rods
8	1	3	3	1

	1 rod	2 rods	3 rods	4 rods	5 rods
16	1	4	6	4	1