

# 4.1 Multiplying and Dividing Rational Expressions

EQ: How do you simplify the multiplication and division of rational expressions by factoring?

Example 1: Simplify the following.

a)  $\frac{(x+1)(x-5)}{(x-5)(x^2-1)}$

$$\frac{\cancel{(x+1)}\cancel{(x-5)}}{\cancel{(x-5)}(x-1)\cancel{(x+1)}} = \frac{1}{x-1}$$

$x \neq 5, 1, -1$

b)  $\frac{x^2+x-12}{x^2+7x+12}$

$$\frac{\cancel{(x+4)}\cancel{(x-3)}}{\cancel{(x+4)}(x+3)} = \frac{x-3}{x+3}$$

$x \neq -4, -3$

You Try! Simplify the following.

a)  $\frac{x^2+6x+9}{x^2-9}$

$$\frac{\cancel{(x+3)}(x+3)}{\cancel{(x+3)}(x-3)} = \frac{x+3}{x-3}$$

$x \neq -3, 3$

b)  $\frac{4x^2+8x}{x^2+6x+8}$

$$\frac{4x\cancel{(x+2)}}{(x+4)\cancel{(x+2)}} = \frac{4x}{x+4}$$

$x \neq -4, -2$

## Multiplying Rational Expressions

When multiplying rational functions, meaning you are multiplying two fractions together, you multiply straight across the top and straight across the bottom, simplifying (i.e. canceling, like we did above) where you can.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Example 2: Simplify completely.

$$\frac{x^2+2x-8}{x^2+4x+3} \cdot \frac{3x+3}{x-2}$$

$$\frac{\cancel{(x+4)}\cancel{(x-2)}(3)\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x+1)}\cancel{(x-2)}}$$

$\frac{3(x+4)}{x+3} \quad x \neq -3, -1, 2$

not crossed out

Example 3: Simplify Completely.

$$\frac{x^2-9}{x^2+5x+6} \cdot \frac{x+2}{3x-9}$$

$$\frac{\cancel{(x+3)}\cancel{(x-3)}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(x+3)}3\cancel{(x-3)}}$$

$$\frac{1}{3} \quad x \neq \pm 3, -2$$

You Try! Multiply the following and state the restrictions.

a)  $\frac{t^2+19t+84}{4t-4} \cdot \frac{2t-2}{t^2+9t+14}$

$$\frac{\cancel{(t+12)}\cancel{(t+7)}2\cancel{(t-1)}}{4\cancel{(t-1)}\cancel{(t+7)}(t+2)} = \frac{2(t+12)}{4(t+2)}$$

$$\frac{(t+12)}{2(t+2)} \quad t \neq -7, -2, 1$$

b)  $\frac{x^2+x-6}{x-5} \cdot \frac{x^2-25}{x^2+4x+3}$

$$\frac{\cancel{(x+3)}\cancel{(x-2)}\cancel{(x+5)}\cancel{(x-5)}}{\cancel{(x-5)}\cancel{(x+3)}(x+1)}$$

$$\frac{(x-2)}{(x+1)}$$

### Dividing Rational Expressions

When dividing rational functions, you multiply the first fraction by the reciprocal of the second fraction, simplifying (i.e. canceling, like we did above) where you can. We call this **SAME-CHANGE-FLIP!**

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

**Example 1:** Simplify completely and state the restrictions.

$$\frac{a+2}{a+3} \div \frac{a^2+a-12}{a^2-9} = \frac{(a+2)(a+3)(a-3)}{(a+3)(a+4)(a-3)} = \frac{a+2}{a+4} \quad a \neq \pm 3, -4$$

**Example 2:** Simplify Completely. State all restrictions.

$$\frac{b^2}{25a-b^2} \cdot \frac{5a-b}{b} = \frac{b^2(5a-b)}{b(5a-b)(5a+b)} = \frac{b}{5a+b} \quad b \neq 0, 5a, -5a$$

$$a \neq \pm \frac{b}{5}$$

**You Try!** Divide the following. Be sure to state all restrictions.

a) 
$$\frac{-12b+18}{b^2-25} \div \frac{b^2-3b-10}{4b-6} = -3 \quad b \neq \pm 5, 3/2$$

b) 
$$\frac{3x+12}{2x+4} \div \frac{x^2-16}{x+2} \cdot \frac{x+2}{x^2-16} = \frac{3(x+4)(x+2)}{2(x+2)(x+4)(x-4)} = \frac{3}{2(x-4)}$$

# Homework 4.1: Multiplying & Dividing Rationals

Honors Math 3

Name: \_\_\_\_\_

Directions: Simplify each rational expression. State any restrictions on the variables.

1.  $\frac{4x+6}{2x+3} = \frac{2(2x+3)}{(2x+3)}$

2.  $\frac{2y}{y^2+6y} = \frac{2y}{y(y+6)}$

3.  $\frac{20+40x}{20x} = \frac{20(2x+1)}{20x}$

2  $x \neq -3/2$

$\frac{2}{y+6}$   $y \neq 0, -6$

$\frac{2x+1}{x}$   $x \neq 0$

4.  $\frac{7x-28}{x^2-16} = \frac{7(x-4)}{(x+4)(x-4)}$

5.  $\frac{3x^2-12}{x^2-x-6} = \frac{3(x-2)(x+2)}{(x-3)(x+2)}$

6.  $\frac{x^2+13x+40}{x^2-2x-35} = \frac{(x+5)(x+8)}{(x-7)(x+5)}$

$\frac{7}{x+4}$   $x \neq \pm 4$

$\frac{3(x-2)}{(x-3)}$   $x \neq 3, -2$

$\frac{x+8}{x-7}$   $x \neq 7, -5$

Directions: Multiply. State any restrictions on the variables.

$\frac{2x+4}{10x} \cdot \frac{15x^2}{x+2} = \frac{30x^2(x+2)}{10x(x+2)}$

8.  $\frac{x^2-5x}{x^2+3x} \cdot \frac{x+3}{x-5} = \frac{x(x-5)(x+3)}{x(x+3)(x-5)}$

$3x$   $x \neq 0, -2$

1  $x \neq 0, -3, 5$

9.  $\frac{x^2-6x}{x^2-36} \cdot \frac{x+6}{x^2} = \frac{x(x-6)(x+6)}{x^2(x+6)(x-6)}$

10.  $\frac{x-2}{(x+2)^2} \cdot \frac{x+2}{2x-4} = \frac{(x-2)(x+2)}{(x+2)(x+2)2(x-2)}$

$\frac{1}{x}$   $x \neq 0, \pm 6$

$\frac{1}{2(x+2)}$   $x \neq \pm 2$

11.  $\frac{3x^3}{x^2-25} \cdot \frac{x^2+6x+5}{x^2} = \frac{3x^3(x+5)(x+1)}{x^2(x+5)(x-5)}$

12.  $\frac{y^2-2y}{y^2+7y-18} \cdot \frac{y^2-81}{y^2-11y+18} = \frac{y(y-2)(y+9)(y-9)}{(y+9)(y-2)(y-3)(y-6)}$

$\frac{3x(x+1)}{x-5}$   $x \neq \pm 5, 0$

$\frac{y}{y-2}$



Directions: Divide. State any restrictions on the variables.

$$\frac{\frac{2x^4}{24y^5} + \frac{21x}{12y^4}}{\frac{84x^4y^4}{504xy^5}}$$

$$\frac{x^3}{6y} \quad x \neq 0 \quad y \neq 0$$

$$\frac{3y+3}{6y+12} \div \frac{18}{5y+5}$$

$$\frac{15(y+1)^2}{108(y+2)} = \frac{5(y+1)^2}{36(y+2)} \quad y \neq -2$$

$$15. \frac{y^2-49}{(y-7)^2} \div \frac{5y+35}{y^2-7y}$$

$$\frac{(y-7)(y+7)}{(y-7)^2} \cdot \frac{y(y-7)}{5(y+7)}$$

$$16. \frac{x^2+10x+16}{x^2-6x-16} \div \frac{x+8}{x-8}$$

$$\frac{(x+8)(x+2)}{(x-8)(x+2)} \cdot \frac{(x-8)}{(x-8)}$$

$$x+8 \quad x \neq \pm 8, -2$$

$$\frac{y}{5} \quad y \neq \pm 7$$

$$17. \frac{y^2-5y+4}{y^2-1} \div \frac{y^2-9}{y^2+5y+4}$$

$$\frac{(y-4)(y-1)(y+4)(y+1)}{(y-1)(y+1)(y-3)(y+3)}$$

$$\frac{(y-4)(y+4)}{(y-3)(y+3)}$$

$$y \neq \pm 3, \pm 1, \pm 4$$

$$18. \frac{x^2-4}{x^2+6x+9} \div \frac{x^2+4x+4}{x^2-9}$$

$$\frac{(x-2)(x+2)}{(x+3)(x+3)} \cdot \frac{(x+3)(x-3)}{(x+2)(x+2)}$$

$$\frac{(x-2)(x-3)}{(x+2)(x+3)} \quad x \neq \pm 3, -2$$

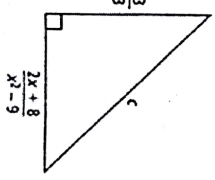
19. What is the area of the triangle shown at the right?

(F)  $\frac{2x+8}{x^2-6x+9}$

(G)  $\frac{x^2+6x+9}{x+4}$

(H)  $\frac{x+4}{x^2-6x+9}$

(I)  $\frac{2x^2+12x+18}{x+4}$



$$A = \frac{1}{2} \left( \frac{2(x+4)}{(x+5)(x-3)} \right) \cdot \frac{(x+3)}{(x-3)} = \frac{(x+4)}{(x-3)(x+3)} = \frac{x+4}{x^2-6x+9}$$

20. What is the quotient  $\frac{y+2}{2y^2-3y-2} \div \frac{y^2-4}{y^2+y-6}$  expressed in simplest form? State any restrictions on the variable.

$$\frac{(y+2)(y+3)(y-2)}{(2y+1)(y-2)(y-2)} = \frac{y+3}{(2y+1)(y-2)}$$

$$y \neq -\frac{1}{2}, \pm 2$$

# 4.2 Adding and Subtracting Rational Expressions

EQ: How do you simplify the addition and subtraction of rational expressions?

In order to add or subtract fractions, we must first find the Least Common Denominator

Arithmetic of Rational Numbers	Arithmetic of Rational Expressions
$a) \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$	$a) \frac{(x-1)}{(x+1)} + \frac{4}{(x-1)} = \frac{3x-3+4x+4}{(x+1)(x-1)} = \frac{7x+1}{(x+1)(x-1)}$
$b) \frac{3}{8} + \frac{5}{4} = \frac{3+10}{8} = \frac{13}{8}$	$b) \frac{(x-1)}{(x+3)} + \frac{4x}{(x-1)(x+3)} = \frac{2x^2-2x+4x}{(x-1)(x+3)} = \frac{2x^2+2x}{(x-1)(x+3)}$
$c) \frac{3}{8} - \frac{1}{6} = \frac{4}{24} = \frac{1}{6}$	$c) \frac{(x-1)}{(x+3)} - \frac{4}{(x-1)} = \frac{2x^2-2x-4x-12}{(x-1)(x+3)} = \frac{2x^2-6x-12}{(x-1)(x+3)}$

### Monomial Denominators-FIND A COMMON DENOMINATOR!

- determine what each denominator has that the other denominator is missing
- multiply top and bottom by whatever is missing-to give you the common denominator

Example 1:  $\frac{1}{6x} + \frac{2}{3x} - \frac{3}{4x}$  CD:  $12x$

$$\frac{2(1)}{2(6x)} + \frac{4(2)}{4(3x)} - \frac{3(3)}{3(4x)}$$

$$\frac{2+8-9}{12x} = \frac{1}{12x} \quad x \neq 0$$

Example 2:  $\frac{5y+2}{xy^2} + \frac{2x-4}{4xy}$  CD:  $4xy^2$

$$\frac{4(5y+2)}{4(xy^2)} + \frac{(2x-4)y}{(4xy)y}$$

$$\frac{20y+8+2xy-4y}{4xy^2} = \frac{2xy+16y+8}{4xy^2}$$

Example 3:  $\frac{3}{7x^2y} + \frac{4}{21xy^2}$  CD:  $21x^2y^2$

$$\frac{(3)3y}{(7x^2y)3y} + \frac{(4)x}{(21xy^2)x}$$

$$\frac{9x}{21x^2y^2} \quad x \neq 0 \quad y \neq 0$$

Example 4:  $\frac{3}{8x^3y^3} - \frac{1}{4xy}$  CD:  $8x^3y^3$

$$\frac{3}{8x^3y^3} - \frac{(1)2x^2y^2}{(4xy)2x^2y^2}$$

$$\frac{3-2x^2y^2}{8x^3y^3}$$

**Bi/Tinomial Denominators - FACTOR & FIND A COMMON DENOMINATOR!**

- Always start by factoring polynomial denominators
- Multiply top and bottom by whatever is missing and then combine the numerators

Example 5:  $\frac{w+12}{4w-16} - \frac{w+4}{2w-8}$  CD:  $4(w-4)$

$$\frac{w+12}{4(w-4)} - \frac{(w+4)2}{(2(w-4))2} = \frac{w+12-2w-8}{4(w-4)} = \frac{-w+4}{4(w-4)} = \frac{-1(\cancel{w-4})}{4(w-4)} = -\frac{1}{4} \quad w \neq 4$$

Example 6:  $\frac{y}{2y+4} - \frac{3}{y+2}$  CD:  $2(y+2)$

$$\frac{y}{2(y+2)} - \frac{(3)2}{(y+2)2} = \frac{y-6}{2(y+2)} \quad y \neq -2$$

Example 7:  $\frac{-3x}{x^2-9} + \frac{4}{2x-6}$  CD:  $2(x-3)(x+3)$

$$\frac{-3x}{x^2-9} + \frac{4}{2x-6} = \frac{2(-3x)}{2(x-3)(x+3)} + \frac{4(x+3)}{2(x-3)(x+3)}$$

$$\frac{-6x+4x+12}{2(x-3)(x+3)} = \frac{-2x+12}{2(x-3)(x+3)} = \frac{-2(x-6)}{2(x-3)(x+3)}$$

Example 8:  $\frac{2x}{x^2-x-2} - \frac{4x}{x^2-3x+2}$  CD:  $(x+1)(x-2)$

$$\frac{2x}{x^2-x-2} - \frac{4x}{x^2-3x+2} = \frac{2x}{(x-2)(x+1)} - \frac{4x}{(x-2)(x+1)}$$

$$\frac{2x^2-2x-4x^2-4x}{(x+1)(x-2)(x-1)} = \frac{-2x^2-6x}{(x+1)(x-2)(x-1)}$$

Example 9:  $\frac{5x}{x^2-x-6} - \frac{4}{x^2+4x+4}$  CD:  $(x-3)(x+3)$

$$\frac{5x}{(x+2)(x+2)} - \frac{4}{(x-3)(x+3)}$$

Example 10:  $\frac{x}{x-1} + \frac{2x-1}{x^2-3x+2}$  CD:  $(x+1)(x-2)$

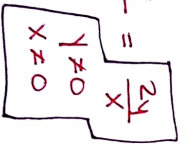
$$\frac{x}{x-1} + \frac{2x-1}{(x+1)(x-2)}$$

$$\frac{x^2-2x+2x-1}{(x-2)(x-1)} = \frac{x^2-1}{(x-2)(x-1)} = \frac{(x+1)(x-1)}{(x-2)(x-1)}$$

**Simplifying Complex Fractions**

- Multiply the top fraction by the reciprocal (flip) of the denominator fraction

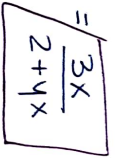
Example 11:  $\frac{\frac{2}{x}}{\frac{1}{y}}$   $\frac{2}{x} \cdot \frac{y}{1} = \frac{2y}{x}$



Example 12:  $\frac{\frac{3}{x+y}}{4}$   $\frac{3}{x+y} \cdot \frac{1}{4} = \frac{3}{4(x+y)}$



Example 13:  $\frac{\frac{3}{2+y}}{\frac{x}{2+y}}$   $\frac{3}{2+y} \cdot \frac{2+y}{x} = \frac{3}{x}$



Example 14:  $\frac{\frac{x-4}{2}}{1-\frac{x-4}{x-4}}$

$$\frac{x^{-4} \cdot \frac{1}{1} - \frac{2}{x-4}}{x^{-4}}$$



# Homework 4.2: Adding & Subtracting Rationals

Name: \_\_\_\_\_

Honors Math 3

Directions: Simplify each sum or difference. State any restrictions on the variables.

1.  $\frac{6y-4}{y^2-5} + \frac{3y+1}{y^2-5}$  CD:  $y^2-5$

2.  $\frac{2y+1}{3y} + \frac{5y+4}{3y}$  CD:  $3y$

3.  $\frac{x^2}{5} + \frac{x^2}{5}$  CD:  $5$

$\frac{3(3y-1)}{y^2-5}$   $y \neq \pm\sqrt{5}$

$\frac{7y+5}{3y}$   $y \neq 0$

$\frac{2x^2}{5}$  No Restrictions

4.  $\frac{3}{8x^3y^3} - \frac{1}{4xy}$  CD:  $8x^3y^3$

5.  $\frac{6}{5x^2y} + \frac{5}{10xy^2}$  CD:  $10x^2y^2$

6.  $\frac{12}{xy^3} - \frac{9}{xy^3}$  CD:  $xy^3$

$\frac{3-2x^2y^2}{8x^3y^3}$   $x \neq 0$   
 $y \neq 0$

$\frac{12y+5x}{10x^2y^2}$   $x \neq 0$   
 $y \neq 0$

$\frac{3}{xy^3}$   $x \neq 0$   
 $y \neq 0$

Directions: Simplify each sum or difference. State any restrictions on the variables.

7.  $\frac{-2}{n+4} - \frac{n^2}{n^2-16}$  CD:  $(n+4)(n-4)$

8.  $\frac{x+2}{x^2+4x+4} + \frac{2}{x+2}$  CD:  $(x+2)$

$\frac{-1(n-2)}{n-4}$   $n \neq \pm 4$

$\frac{3}{x+2}$   $x \neq -2$

9.  $\frac{4}{x^2-25} + \frac{6}{x^2+6x+5}$  CD:  $(x+5)(x-5)(x+1)$

10.  $\frac{y}{4y+8} - \frac{1}{y^2+2y}$  CD:  $4y(y+2)$

$\frac{2(5x-13)}{(x+5)(x-5)(x+1)}$   $x \neq \pm 5, -1$

$\frac{y-2}{4y}$   $y \neq -2, 0$

Directions: Simplify the complex fractions. State any restrictions on the variables.

11.  $\frac{\frac{2}{3}}{\frac{x}{y}}$   $\frac{2y}{3x}$   $x \neq 0$   
 $y \neq 0$

12.  $\frac{1+\frac{2}{x}}{4-\frac{6}{x}}$   $\frac{x+2}{2(2x-3)}$   $x \neq \frac{3}{2}, 0$

$\frac{\frac{3}{x+1}}{\frac{5}{x-1}}$   $\frac{3(x-1)}{5(x+1)}$   $x \neq \pm 1$

14.  $\frac{\frac{x^2-1}{3}}{x+1}$

15. Angelic simplified the following rational expressions. She correctly simplified one out of the three problems. Determine which one she answered correctly. Also, identify and correct where she went wrong in the other two problems.

a.  $\frac{5x}{(x-3)} + \frac{2}{(x-1)}$   $\frac{5x(x-1)}{(x-3)(x-1)} + \frac{2(x-3)}{(x-3)(x-1)}$   
 b.  $\frac{x}{(x+3)} - \frac{4(x+3)}{(x-1)}$   $\frac{x}{1} - \frac{4}{(x-1)}$   
 c.  $\frac{(x+1)(x-2)}{(x+2)} \times \frac{(x+5)}{(x-2)(x+2)}$   $\frac{(x+1)(x-2)(x+5)}{(x+2)(x-2)(x+2)}$

Forgot to Distribute 5x to the -1  
 5x to the -1  
 a)  $\frac{7x-11}{(x-3)(x-1)}$  ← correct answer  
 b)  $\frac{x^2-5x+2}{(x+3)(x-1)}$  ← correct answer  
 Didn't multiply  $\frac{x}{x+3}$  by  $(x-1)$  and they canceled  
 C is correct!

16. Which is the least common denominator of fractions that have denominators  $5x + 10$  and  $25x^2 - 100$ ?  
 (A)  $5(x-2)$  (B)  $5(x^2-20)$   
 (C)  $25(x^2-4)$  (D)  $75(x+2)(x^2-4)$   
 $5(x+2)$   
 $25(x^2-4) = 25(x+2)(x-2)$

17. Which expression equals  $\frac{\frac{x}{y} + 6}{\frac{1}{y}}$ ?  
 (A)  $\frac{12y}{x}$  (B)  $\frac{2y+6xy}{x}$  (C)  $\frac{6x+2}{xy}$  (D)  $\frac{2y+x}{6xy}$   
 $\frac{\frac{x}{y} + 6}{\frac{1}{y}} = \frac{2+6x}{x} = \frac{2+6xy}{x}$

18. Which expression equals  $\frac{4}{x^2-3x} + \frac{6}{3x-9}$ ?  
 (A)  $\frac{2(x+2)}{x(x-3)}$  (B)  $\frac{10}{x^2-9}$  (C)  $\frac{4x+18}{3x(x-3)}$  (D)  $\frac{2}{x}$

$\frac{4(3)}{3x(x-3)} + \frac{6x}{3x(x-3)} = \frac{12+6x}{3x(x-3)} = \frac{2(x+2)}{x(x-3)}$

19. Subtract  $3 - \frac{1}{x^2+5}$ . Write your answer in simplest form. State any restrictions on the variable.  
 $\frac{3(x^2+5)-1}{x^2+5} = \frac{3x^2+15-1}{x^2+5} = \frac{3x^2+14}{x^2+5}$



# 4.4 Solving Rational Equations

## How to Solve Rational Equations

1. Find a common denominator on each side of the equation
2. Simplify each side into one term, and then cross multiply.
3. Solve for the variable.
4. Check your answers for extraneous solutions!

Example 1: Solve  $\frac{(1)}{5} + \frac{(5)}{2} = 7$   
 $\frac{(2)}{10} + \frac{(5)}{2} = 7$

$$\frac{2y + 5y}{10} = 7$$

$$\frac{7y}{10} = 7$$

$$7y = 70$$

$$y = 10$$

Example 2: Solve  $\frac{11}{3x} - \frac{1}{3} = \frac{-4}{x^2}$

$$\frac{11-x}{3x} = \frac{-4}{x^2}$$

$$x^2(-x) = -4(3x)$$

$$11x^3 = -12x$$

$$x^3 - 11x^2 - 12x = 0$$

Example 3: Solve  $\frac{4}{x+2} + 3 = \frac{3x}{x-3}$

$$\frac{4+3(x+2)}{x+2} = \frac{3x}{x-3}$$

$$4+3x+6 = \frac{3x}{x-3}$$

$$10+3x = \frac{3x}{x-3}$$

$$(10+3x)(x-3) = 3x$$

$$10x - 30 + 3x^2 - 9x = 3x$$

$$3x^2 + 10x - 9x - 30 = 3x^2 + 6x - 30$$

$$3x^2 + 4x - 30 = 3x^2 + 6x - 30$$

$$-2x = 0$$

$$x = 0$$

Example 4: Solve  $\frac{1}{x-2} - 7 = \frac{1}{x-2}$

You Try! Solve  $\frac{3x-2}{12} - \frac{1}{6} = \frac{1}{6}$

$$\frac{3x-2-2}{12} = \frac{1}{6}$$

$$\frac{3x-4}{12} = \frac{1}{6}$$

$$3x-4 = 2$$

$$3x = 6$$

$$x = 2$$

You Try! Solve  $\frac{3x-2}{12} - \frac{1}{6} = \frac{1}{6}$

$$\frac{3x-2-2}{12} = \frac{1}{6}$$

$$\frac{3x-4}{12} = \frac{1}{6}$$

$$3x-4 = 2$$

$$3x = 6$$

$$x = 2$$

You Try! Solve  $\frac{7x-12}{4x^2} = \frac{1}{2x^2}$

$$7x-12 = \frac{1}{2}$$

$$14x-24 = 1$$

$$14x = 25$$

$$x = \frac{25}{14}$$

You Try! Solve  $\frac{5+2(x-4)}{(x-4)} = \frac{2x}{x-3}$

$$(5+2(x-4))(x-3) = 2x(x-4)$$

$$(5+2x-8)(x-3) = 2x(x-4)$$

$$(-3+2x)(x-3) = 2x(x-4)$$

$$-3x+9+2x^2-6x = 2x^2-8x$$

$$-9x+9 = -8x$$

$$-x = -9$$

$$x = 9$$

You Try! Solve  $\frac{1}{x-4} = 4 + \frac{1}{x-4}$

$$\frac{1}{x-4} - 4 = \frac{1}{x-4}$$

$$\frac{1-4(x-4)}{x-4} = \frac{1}{x-4}$$

$$1-4x+16 = 1$$

$$-4x+17 = 1$$

$$-4x = -16$$

$$x = 4$$

You Try! Solve  $\frac{1}{x-4} = 4 + \frac{1}{x-4}$

$$\frac{1}{x-4} - 4 = \frac{1}{x-4}$$

$$\frac{1-4(x-4)}{x-4} = \frac{1}{x-4}$$

$$1-4x+16 = 1$$

$$-4x+17 = 1$$

$$-4x = -16$$

$$x = 4$$

Example 5: Solve  $\frac{3}{x-3} - \frac{7}{x+3} = \frac{2}{x^2-9}$

$$\frac{(x+3) - 7(x-3)}{(x+3)(x-3)} = \frac{2}{(x+3)(x-3)}$$

$$3x + 9 - 7x + 21 = 2$$

$$-4x + 30 = 2$$

$$-4x = -28$$

$$\boxed{x = 7}$$

Example 6: Solve  $\frac{x}{x-4} = \frac{15}{x-3} - \frac{2x}{x^2-7x+12}$

$$\frac{x}{x-4} - \frac{15}{x-3} = \frac{-2x}{(x-4)(x-3)}$$

$$\frac{x(x-3) - 15(x-4)}{(x-4)(x-3)} = \frac{-2x}{(x-4)(x-3)}$$

$$x^2 - 3x - 15x + 60 = -2x$$

$$x^2 - 18x + 60 = -2x$$

$$x^2 - 16x + 60 = 0$$

$$(x-10)(x-6) = 0$$

$$\boxed{x = 10, 6}$$

Directions: Decide whether each of the following is an expression or an equation. If it is an equation, solve it. If it is an expression, write it as a single fraction.

1.  $\frac{5x}{2y+4} - \frac{6}{y^2+2y}$  Expression

2.  $\frac{1}{2x} - \frac{2}{5x} = \frac{1}{2}$  Equation

$$\frac{5x}{2(4+2)y} + \frac{-6(2)}{y(4+2)(2)}$$

$$\frac{5xy - 12}{2y(4+2)}$$

$$\boxed{x = \frac{1}{5}}$$

3.  $3x - \frac{x^2-5x}{x^2-2}$  Expression

$$\frac{3x(x^2-2) - (x^2-5x)}{x^2-2}$$

$$3x^3 - 6x - x^2 + 5x$$

$$\frac{3x^3 - x^2 - x}{x^2-2}$$

4.  $\frac{-2}{x^2-2} = \frac{2}{x-4}$  Equation

$$-2(x-4) = 2(x^2-2)$$

$$-2x + 8 = 2x^2 - 4$$

$$2x^2 + x - 12 = 0$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$\boxed{x = -3, x = 2}$$

You Try! Solve  $\frac{4}{x-4} - \frac{3}{x+1} = \frac{5}{x^2-3x-4}$

$$\frac{4(x+1) - 3(x-4)}{(x-4)(x+1)} = \frac{5}{(x-4)(x+1)}$$

$$4x + 4 - 3x + 12 = 5$$

$$x + 16 = 5$$

$$\boxed{x = -11}$$

You Try! Solve  $\frac{3x}{x+2} - \frac{2}{x+3} = \frac{36}{x^2+5x+6}$

# Homework 4.4: Solving Rational Equations

Honors Math 3

Name: \_\_\_\_\_

Directions: Decide whether each of the following is an expression or an equation. If it is an equation, solve it. If it is an expression, write it as a single fraction.

1.  $\frac{x}{4} - \frac{x}{7} = 3$  equation

$$\frac{7x - 4x}{28} = \frac{3}{1}$$

$$3x = 84$$

$$x = 28$$

2.  $\frac{x}{2} - \frac{x}{3}$  expression

$$\frac{5x - 2x}{10} = \frac{3x}{10}$$

3.  $\frac{x}{6} - \frac{x}{8}$  expression

$$\frac{4x - 3x}{24} = \frac{x}{24}$$

4.  $\frac{3x+1}{4} = \frac{x-1}{1}$  equation

$$3x+1 = 4(x-1)$$

$$3x+1 = 4x-4$$

$$-x+1 = -4$$

$$-x = -5$$

$$x = 5$$

5.  $\frac{3x-1}{2} - \frac{x-x+3}{5} - \frac{x+3}{4}$  expression

$$\frac{10(3x-1) - 4(x) - 5(x+3)}{20}$$

$$\frac{30x-10-4x-5x-15}{20}$$

$$\frac{21x-25}{20}$$

Directions: Solve each equation. Check for extraneous solutions.

7.  $\frac{4}{x} + \frac{3}{4} = \frac{10}{x}$

$$\frac{4(4) + 3(x)}{4x} = \frac{10}{x}$$

$$(16 + 3x)(x) = 4x(10)$$

$$3x^2 + 16x = 40x$$

$$\frac{5}{x-2} = \frac{4}{x-1}$$

$$4(x-2) = 5(x-1)$$

$$4x-8 = 5x-5$$

$$-x-8 = -5$$

$$-x = 3 \quad x = -3$$

9.  $\frac{9}{x} + 2 = \frac{2x}{x+3}$

$$\frac{9(x+3) + 2x(x+3)}{x(x+3)} = \frac{2x^2}{x(x+3)}$$

$$2x^2 + 6x + 9x + 27 = 2x^2$$

$$15x + 27 = 0$$

$$15x = -27$$

$$x = -1.8$$

$$\frac{6+3x}{x} = \frac{3x}{x+1}$$

$$(3x+6)(x+1) = 3x(x)$$

$$3x^2 + 3x + 6x + 6 = 3x^2$$

$$9x + 6 = 0$$

$$9x = -6$$

$$x = -\frac{2}{3}$$

11.  $\frac{3}{x+2} - \frac{5}{x} = \frac{13}{x+2}$

$$\frac{3x-5(x+2)}{x(x+2)} = \frac{13(x)}{(x+2)(x)}$$

$$3x-5x-10 = 13x$$

$$-2x-10 = 13x$$

$$-10 = 15x$$

$$x = -\frac{2}{3}$$

$$\frac{7}{x} - \frac{2}{x-3} = \frac{6}{x}$$

$$7(x-3) - 2(x) = 6(x-3)$$

$$7x-21-2x = 6x-18$$

$$5x-21 = 6x-18$$

$$-x-3 = -3$$

$$-x = 0$$

$$x = 0$$



$$13. \frac{3}{2} + \frac{2}{2x-4} = \frac{1}{x-2}$$

$$\frac{3(x-2) + 2}{2(x-2)} = \frac{1(2)}{2(x-2)}$$

$$3x - 6 + 2 = 2$$

$$3x - 4 = 2$$

$$3x = 6$$

$$x = 2$$

[No Solution]

$$15. \frac{x}{3x+12} + \frac{x-1}{x+4} = \frac{5}{3}$$

$$\frac{x + 3(x-1)}{3(x+4)} = \frac{5}{3}$$

$$-3x = 69$$

$$x = -23$$

$$\frac{x + 3x - 3}{3x + 12} = \frac{5}{3}$$

$$3(-1x - 3) = 5(3x + 12)$$

$$-3x - 9 = 15x + 60$$

$$17. \frac{1}{x-2} - \frac{2}{x+2} = \frac{2}{x^2-4}$$

$$\frac{1(x+2) - 2(x-2)}{(x-2)(x+2)} = \frac{2}{(x-2)(x+2)}$$

$$x + 2 - 2x + 4 = 2$$

$$-x + 6 = 2$$

$$-x = -4$$

$$x = 4$$

$$19. \frac{x}{x-3} - 2 = \frac{3}{x-3}$$

$$\frac{x - 2(x-3)}{x-3} = \frac{3}{x-3}$$

$$x - 2x + 6 = 3$$

$$x + 6 = 3$$

$$-x = -3$$

$$x = 3$$

[No Solution]

$$14. \frac{2}{x-1} + \frac{5}{2x-2} = \frac{3}{4}$$

$$36 = 6x - 6$$

$$42 = 6x$$

$$x = 7$$

$$\frac{2(2) + 5}{2(x-1)} = \frac{3}{4}$$

$$\frac{9}{2x-2} = \frac{3}{4}$$

$$36 = 3(2x-2)$$

$$16. \frac{x+1}{x-2} - \frac{x+3}{x} = \frac{6}{x^2-2x}$$

$$\frac{x(x+1) - (x+3)(x-2)}{x(x-2)} = \frac{6}{x(x-2)}$$

$$x^2 + x - (x^2 + x - 6) = 6$$

$$x^2 + x - x^2 - x + 6 = 6$$

$$6 = 6$$

[∞ many solutions  
x ≠ 0, 2]

$$18. \frac{1}{x+4} + \frac{1}{x-4} = \frac{12}{x^2-16}$$

$$\frac{(x-4) + (x+4)}{(x+4)(x-4)} = \frac{12}{(x-4)(x+4)}$$

$$2x = 12$$

$$x = 6$$

$$20. \frac{x}{x-5} + 2 = \frac{5}{x-5}$$

$$\frac{x + 2(x-5)}{x-5} = \frac{5}{x-5}$$

$$x + 2x - 10 = 5$$

$$3x = 15$$

$$x = 5$$

[No Solution]

# 4.5 Graphing Rational Expressions

Q: How do you graph rational expressions, state points of discontinuity, and find any horizontal or vertical asymptotes?

**Example 1:** Simplify the following. State any restrictions on the variables.

a)  $\frac{(x+1)(x-5)}{(x-5)(x^2-1)} = \frac{\cancel{(x+1)}\cancel{(x-5)}}{\cancel{(x-5)}(x+1)(x-1)} = \frac{1}{x-1}$       b)  $\frac{x^2+x-12}{x^2+7x+12} = \frac{\cancel{(x+4)}(x-3)}{\cancel{(x+4)}(x+3)} = \frac{x-3}{x+3}$

$x \neq 5, \pm 1$        $x \neq -4, -3$

\* x-int are any quantities that are left in the numerator. Vertical Asymptotes: Where the denominator of a function equals zero. \* Point of Discontinuity: A hole in the graph.

**Example 2:** Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of  $f(x) = \frac{x^2-1}{x^2-6x+5}$ .

$\frac{(x-1)(x+1)}{(x-2)(x-3)}$       Holes: None  
 VA:  $x = 2, 3$   
 x-int:  $(0, 1), (0, -1)$

**Example 3:** Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of  $f(x) = \frac{x^2-4}{x^2+5x+6}$ .

$\frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x+3)}$       Holes:  $x = -2$   
 VA:  $x = -3$   
 x-int:  $(0, 2)$

**Horizontal Asymptotes:** determined by comparing the degree of the numerator to the degree of the denominator. Let  $m$  = degree of numerator and  $n$  = degree of denominator.

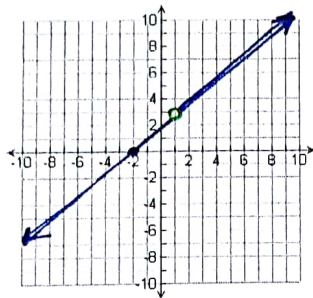
If...	Then the graph has...
<p><math>m &lt; n</math></p> <p><i>Larger degree in denominator</i>  <math>y=0</math></p> <p><math>f(x) = \frac{x+4}{\cancel{(x+4)}(x+1)}</math></p>	<p>A horizontal asymptote at <math>y = 0</math></p> <p>x-int: <u>none</u></p> <p>V.A.: <math>x = -1</math>      Hole(s): <math>x = -4</math></p> <p>H.A.: <math>y = 0</math>      Domain: <math>(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)</math></p>
<p><math>m = n</math></p> <p><i>equal degrees. H.A. is the fraction of the leading coefficients</i></p> <p><math>f(x) = \frac{x^2+5x+4}{4x^2-9}</math></p> <p><math>\frac{(x+4)(x+1)}{(2x-3)(2x+3)}</math></p>	<p>A horizontal asymptote at the coefficient of <math>m</math> divided by the coefficient of <math>n</math></p> <p>x-int: <math>(-4, 0)</math>  <math>(-1, 0)</math></p> <p>V.A.: <math>x = -3/2, 3/2</math>      Hole(s): None</p> <p>H.A.: <math>y = 1/4</math>      Domain: <math>(-\infty, -3/2) \cup (-3/2, 3/2) \cup (3/2, \infty)</math></p>
<p><math>m &gt; n</math></p> <p><i>Larger degree in numerator</i>  no H.A.</p> <p><math>f(x) = \frac{(x+4)\cancel{(x+1)}}{\cancel{x+4}}</math></p>	<p>No horizontal asymptote</p> <p>V.A.: None      Hole(s): <math>x = -4</math></p> <p>H.A.: None      Domain: <math>(-\infty, -4) \cup (-4, \infty)</math></p>



**Example 4:** State the asymptotes and points of discontinuity of each equation, and then graph the function and state the domain.

a)  $f(x) = \frac{x^2 + x - 2}{x - 1}$

$\frac{(x+2)(\cancel{x-1})}{\cancel{x-1}}$



Holes:  $x = 1$

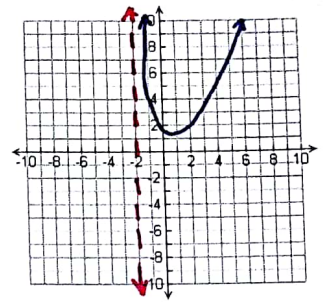
VA: None

HA: None

x-int:  $(-2, 0)$

Domain:  $(-\infty, 1) \cup (1, \infty)$

b)  $f(x) = \frac{2x^2 + 3}{x + 2}$



Holes: None

VA:  $x = -2$

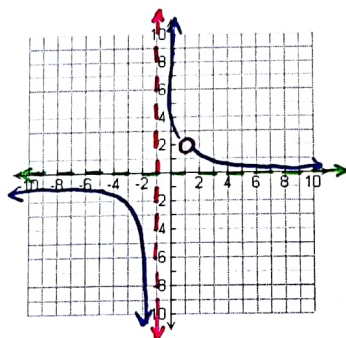
HA: None

x-int: None  
(imaginary)

Domain:  $(-\infty, -2) \cup (-2, \infty)$

c)  $f(x) = \frac{x-1}{x^2-1}$

$\frac{\cancel{x-1}}{(x-1)(x+1)}$



Holes:  $x = 1$

VA:  $x = -1$

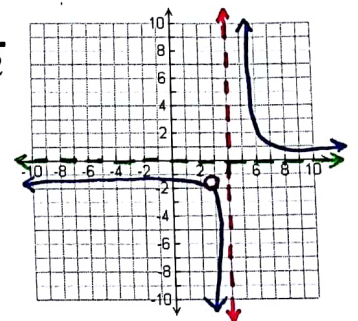
HA:  $y = 0$

x-int: none

Domain:  $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

d)  $f(x) = \frac{x-3}{x^2-7x+12}$

$\frac{\cancel{x-3}}{(x-4)(x-3)}$



Holes:  $x = 3$

VA:  $x = 4$

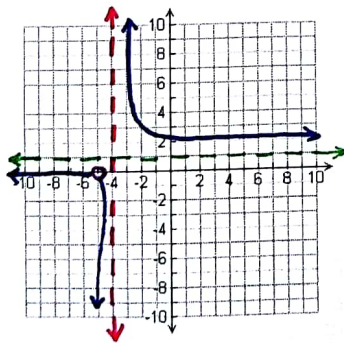
HA:  $y = 0$

x-int: None

Domain:  $(-\infty, 3) \cup (3, 4) \cup (4, \infty)$

e)  $f(x) = \frac{x^2 + 10x + 25}{x^2 + 9x + 20}$

$\frac{(x+5)(x+5)}{(x+5)(x+4)}$



Hole:  $x = -5$

VA:  $x = -4$

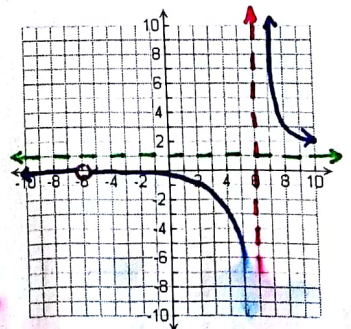
HA:  $y = 1$

x-int:  $x = -5$

Domain:  $(-\infty, -5) \cup (-5, -4) \cup (-4, \infty)$

f)  $f(x) = \frac{x^2 + 12x + 36}{x^2 - 36}$

$\frac{(x+6)(x+6)}{(x+6)(x-6)}$



Hole:  $x = -6$

VA:  $x = 6$

HA:

x-int:



Directions: Identify points of discontinuity, vertical asymptotes, horizontal asymptotes, and x-intercepts. Do not graph the functions.

9.  $f(x) = \frac{1}{3x^2 + 3x - 18}$

Hole: None  
 VA:  $x = -3$   $x = 2$   
 HA:  $y = 0$   
 X-int: none

10.  $f(x) = \frac{x-2}{x-4}$

Hole: None  
 VA:  $x = 4$   
 HA:  $y = 1$   
 X-int:  $(2, 0)$

11.  $f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$

$$\frac{x(x^2 - x - 6)}{-3(x^2 + x - 6)} = \frac{x(x-3)(x+2)}{-3(x+3)(x-2)}$$

Hole: none  
 VA:  $x = -3$   $x = 2$   
 HA: None  
 X-int:  $(0, 0)$   $(3, 0)$   $(-2, 0)$

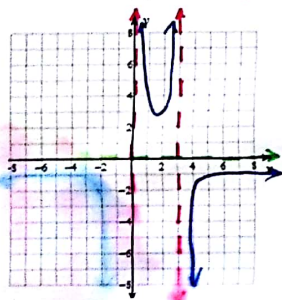
12.  $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$

$$\frac{(x+3)(x-2)}{-4(x^2 + 4x + 3)} = \frac{(x+3)(x-2)}{-4(x+3)(x+1)}$$

Hole:  $x = -3$   
 VA:  $x = -1$   
 HA:  $y = -1/4$   
 X-int:  $(2, 0)$

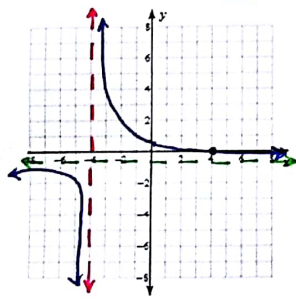
Directions: Identify points of discontinuity, vertical asymptotes, horizontal asymptotes, and x-intercepts. Then, sketch the graph and state the domain.

13.  $f(x) = -\frac{4}{x^2 - 3x} = \frac{-4}{x(x-3)}$



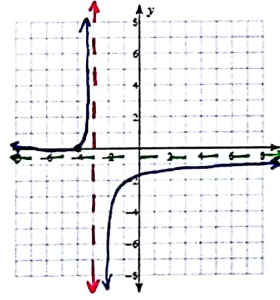
POD:  $(0, 0)$   $(3, 0)$   
 VA:  $x = 0$   $x = 3$   
 HA:  $y = 0$   
 X-int: none  
 Domain:  $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

14.  $f(x) = \frac{x-4}{-4x-16} = \frac{x-4}{-4(x+4)}$



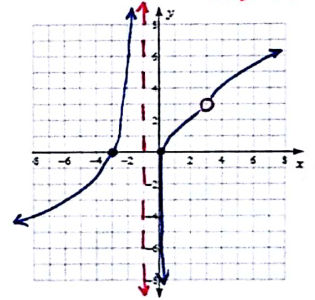
POD: None  
 VA:  $x = -4$   
 HA:  $y = -1/4$   
 X-int:  $(4, 0)$   
 Domain:  $(-\infty, -4) \cup (-4, \infty)$

15.  $f(x) = \frac{x+4}{-2x-6} = \frac{x+4}{-2(x+3)}$



POD: None  
 VA:  $x = -3$   
 HA:  $y = -1/2$   
 X-int:  $(-4, 0)$   
 Domain:  $(-\infty, -3) \cup (-3, \infty)$

16.  $f(x) = \frac{x^3 - 9x}{3x^2 - 6x - 9} = \frac{x(x-3)(x+3)}{3(x-3)(x+1)}$



POD:  $x = 3$   
 VA:  $x = -1$   
 HA: None  
 X-int:  $(-3, 0)$   $(0, 0)$   
 Domain:  $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

# Homework 4.5: Graphing Rational Functions

Honors Math 3

Name: \_\_\_\_\_

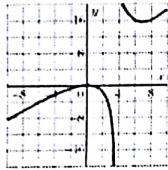
Directions: Answer each of the following multiple choice questions to the best of your ability. Show all necessary work.

1. Which value of  $x$  will make the fraction  $\frac{x-3}{x+6}$  undefined?

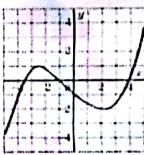
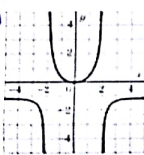
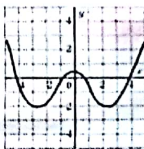
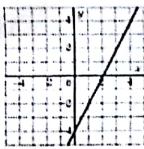
- A. 6    **B.** -6    C. 3    D. -3

2. Which of the following is the equation of an asymptote for the function graphed?

- A.  $x = -4$   
 B.  $y = 0$   
**C.**  $x = 4$   
 D.  $y = 16$



3. Which of the following could be the graph of a rational function that is *not* a polynomial function?

- A.     **C.**   
 B.     D. 

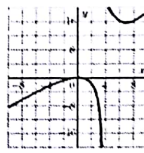
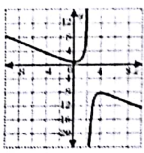
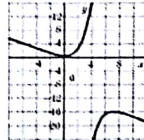
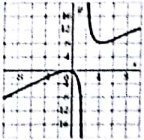
4. The graph of the function  $y = \frac{8}{x}$  lies in what quadrant(s)?

- A.** the first and third quadrants  
 B. the second and fourth quadrants  
 C. the second quadrant only  
 D. the third quadrant only

5. What is the value of the hole for the graph of  $y = \frac{x^2 - 25}{x - 5}$ ?

- A. -5    **B.** 5  
 C. 25    D. There is no hole

6. Which of the following represents the graph of  $y = \frac{x^2}{x-2}$ ?

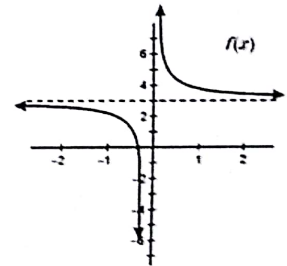
- A.     C.   
 B.     **D.** 

7. What is the domain of the function?

$$f(x) = \frac{8}{x+3} - 2$$

- A. all real numbers except 2  
 B. all real numbers  
 C. all real numbers except 3  
**D.** all real numbers except -3

8. What value(s) are restricted from the range of  $f(x)$ ?



- A. 1    B. 0    **C.** 3  
 D. there are no restricted values

9. A value of  $x$  that makes the expression  $\frac{x^2 + 4x - 12}{x^2 - 2x - 15}$  undefined is

- A. -6    B. -2    C. 3    **D.** 5