3.1 Radical Operations

EQ: How can we use the index to help simplify a radical expression?

radical symbol

Index

$z\sqrt[x]{y}$

Coefficent

radicand

When simplifying radicals, factor the insides of the radical. Then circle the pairs, triples or quadruples (depending on the index) and pull them outside. Multiply outside factors on the outside and inside factors on the inside.

Example 1: Simplify $\sqrt[3]{54y^{10}}$

54
9 6
3 3 3 2

$\sqrt[3]{3\cdot 3\cdot 3\cdot 2\; yyyyyyyyyy}$

$3yyy\sqrt[3]{2y}$

$\boxed{3y^3\sqrt[3]{2y}}$

Example 2: Simplify $\sqrt[4]{32x^6y^2}$

32
8 4
4 2 2 2
2 2

$\sqrt[4]{2\cdot 2\cdot 2\cdot 2\; 2\; yy\; xxxx\; x\; x}$

$\boxed{2x\sqrt[4]{2x^2y^2}}$

Example 3: Simplify $x\sqrt{8x^6}$

8
4 2
2 2

$x\sqrt{2\cdot 2\cdot 2\; xx\, xx\, xx}$

$x\,2\,xxx\sqrt{2}$

$\boxed{2x^4\sqrt{2}}$

Example 4: Simplify $\sqrt[3]{27x^9y^3}$

27
9 3
3 3

$\sqrt[3]{3\cdot 3\cdot 3\; yyy\; xxx\, xxx\, xxx}$

$\boxed{3x^3y}$

When multiplying radicals, start by simplifying the radicals, if necessary. Then, multiply **outsides by outsides** and **insides by insides**. Then simplify again.

Example 5: $\sqrt{40}\cdot\sqrt{20} = \sqrt{800}$

800
40 20
10 4 10 2
2 5 2 2 2 5

$\sqrt{2\cdot 2\; 2\cdot 2\cdot 2\; 5\cdot 5}$

$2\cdot 2\cdot 5\sqrt{2}$

$\boxed{20\sqrt{2}}$

Example 6: $\sqrt{12}\cdot\sqrt{18} = \sqrt{216}$

216
12 18
6 2 6 3
2 3 2 3

$\sqrt{2\cdot 2\; 2\; 3\cdot 3\cdot 3}$

$\boxed{6\sqrt{6}}$

Example 7: $2\sqrt{8}\cdot 3\sqrt{2} = 6\sqrt{16}$

$= 6(4)$

$\boxed{= 24}$

Example 8: $-6\sqrt{10}\cdot\sqrt{15} = -6\sqrt{150}$

150
15 10
5 3 5 2

$-6\sqrt{5\cdot 5\cdot 3\cdot 2}$

$\boxed{-30\sqrt{6}}$

Adding Radicals

**Only add like terms (like radicands and indices!)…so simplify first, then add if possible.

Example 1: $\sqrt{72}+\sqrt{32}$

72: 8, 9; 4, 2, 3, 3; 2, 2

32: 8, 4; 4, 2, 2, 2; 2, 2

$6\sqrt{2} + 4\sqrt{2} = \boxed{10\sqrt{2}}$

Example 2: $2\sqrt{20}+5\sqrt{80}$

$2\sqrt{20} + 5\sqrt{80}$

20: 4, 5; 2, 2

80: 8, 10; 4, 2, 2, 5; 2, 2

$4\sqrt{5} + 20\sqrt{5} = \boxed{24\sqrt{5}}$

Subtracting Radicals

**Only subtract like terms (like radicands and indices!)…so simplify first, then subtract if possible

Example 3: $6\sqrt{18}-3\sqrt{50}$

$6\sqrt{18} - 3\sqrt{50}$

18: 6, 3; 2, 3

50: 5, 10; 5, 2

$18\sqrt{2} - 15\sqrt{2} = \boxed{3\sqrt{2}}$

Example 4: $\sqrt[3]{81}-2\sqrt[3]{24}$

$\sqrt[3]{81} - 2\sqrt[3]{24}$

81: 9, 9; 3, 3, 3, 3

24: 6, 4; 3, 2, 2, 2

$3\sqrt[3]{3} - 4\sqrt[3]{3} = \boxed{-1\sqrt[3]{3}}$

Multiplying Radicals

Multiply using the BOX method or FOIL method

Example 5: $(2\sqrt{3}+3\sqrt{5})(3-\sqrt{3})$

	$2\sqrt{3}$	$3\sqrt{5}$
3	$6\sqrt{3}$	$9\sqrt{5}$
$-\sqrt{3}$	$-2\sqrt{9}$ $=-2(3)$	$-3\sqrt{15}$

$\boxed{-6+6\sqrt{3}+9\sqrt{5}-3\sqrt{15}}$

Example 6: $(4\sqrt{2}+7)^2 = (4\sqrt{2}+7)(4\sqrt{2}+7)$

$(4\sqrt{2}+7)(4\sqrt{2}+7)$

$16\sqrt{4} + 28\sqrt{2} + 28\sqrt{2} + 49$

$16(2) + 56\sqrt{2} + 49$

$32 + 56\sqrt{2} + 49$

$\boxed{81 + 56\sqrt{2}}$

Simplifying Quotients

use a conjugate to rationalize a denominator

conjugate– expressions that differ only in the signs of the second terms. (x + y and x – y are conjugates)

Example 7: $\dfrac{2+\sqrt{3}}{4-\sqrt{3}} \cdot \dfrac{(4+\sqrt{3})}{(4+\sqrt{3})}$

$\dfrac{8+2\sqrt{3}+4\sqrt{3}+3}{16+4\sqrt{3}-4\sqrt{3}-3} = \boxed{\dfrac{11+6\sqrt{3}}{13}}$

Example 8: $\dfrac{1+\sqrt{5}}{2-\sqrt{7}} \cdot \dfrac{(2+\sqrt{7})}{(2+\sqrt{7})}$

$\dfrac{2+1\sqrt{7}+2\sqrt{5}+\sqrt{35}}{4+2\sqrt{7}-2\sqrt{7}-7} = \boxed{\dfrac{2+2\sqrt{5}+1\sqrt{7}+\sqrt{35}}{-3}}$

Homework 3.1: Operations of Radicals

Name: ______________________

Complete #2 – 26 even

Honors

Directions: Simplify the following. No decimal answers accepted. Show all work on a separate sheet of paper

1) $\sqrt{252} = \boxed{6\sqrt{7}}$

2) $\sqrt{24} = \boxed{2\sqrt{6}}$

3) $-8\sqrt{384} = \boxed{-64\sqrt{6}}$

4) $7\sqrt{175} = \boxed{35\sqrt{7}}$

5) $-\sqrt{3}+3\sqrt{3} = \boxed{2\sqrt{3}}$

6) $-3\sqrt{2}-3\sqrt{2} = \boxed{-6\sqrt{2}}$

7) $3\sqrt{3}-\sqrt{3}-2\sqrt{2} = \boxed{2\sqrt{3}-2\sqrt{2}}$

8) $2\sqrt{5}-2\sqrt{6}+3\sqrt{5} = \boxed{5\sqrt{5}-2\sqrt{6}}$

9) $\sqrt{6}+\sqrt{24} = \boxed{3\sqrt{6}}$

10) $\sqrt{5}+\sqrt{5} = \boxed{2\sqrt{5}}$

11) $-2\sqrt{12}-\sqrt{12} = \boxed{-6\sqrt{3}}$

12) $3\sqrt{54}+2\sqrt{24} = \boxed{13\sqrt{6}}$

13) $-\sqrt{6}+2\sqrt{6}-\sqrt{18} = \boxed{\sqrt{6}-3\sqrt{2}}$

14) $3\sqrt{12}-2\sqrt{12}-\sqrt{54} = \boxed{2\sqrt{3}-3\sqrt{6}}$

15) $\sqrt{3}\cdot\sqrt{3} = \boxed{3}$

16) $\sqrt{5}\cdot\sqrt{5} = \boxed{5}$

17) $-3\sqrt{15}(5+\sqrt{3}) = \boxed{-15\sqrt{15}-9\sqrt{5}}$

18) $3\sqrt{5}(\sqrt{5}+3) = \boxed{15+9\sqrt{5}}$

19) $(4\sqrt{5}-3)(\sqrt{5}-2) = \boxed{26-11\sqrt{5}}$

20) $(3\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3}) = \boxed{18+4\sqrt{15}}$

21) $\dfrac{\sqrt{3}}{\sqrt{5}} = \boxed{\dfrac{\sqrt{15}}{5}}$

22) $\dfrac{\sqrt{2}}{\sqrt{5}} = \boxed{\dfrac{\sqrt{10}}{5}}$

23) $\dfrac{4\sqrt{3}}{5\sqrt{5}} = \boxed{\dfrac{4\sqrt{15}}{25}}$

24) $\dfrac{2\sqrt{2}}{4\sqrt{3}} = \boxed{\dfrac{\sqrt{6}}{6}}$

25) $-\dfrac{2}{5+\sqrt{3}} = \boxed{-\dfrac{10-2\sqrt{3}}{22}}$

26) $\dfrac{2}{-2-5\sqrt{2}}$ $\boxed{\dfrac{-2+5\sqrt{2}}{-23} \text{ or } \dfrac{2-5\sqrt{2}}{23}}$

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Honors Math 3

3.2 Imaginary & Complex Numbers

EQ: What is a complex number? How do we simplify complex expressions?

Imaginary Numbers

Until now you have been told you cannot take the square root of a negative number. Now, however, you can take the square root of a negative number, but it involves a new number called "i" which is called the imaginary number.

$$i = \sqrt{-1}$$

Simplify:

Ex. $\sqrt{-25}$

$i\sqrt{25}$

$\boxed{5i}$

Ex. $\sqrt{-9}$

$i\sqrt{9}$

$\boxed{3i}$

Ex. $\sqrt{-40}$

$i\sqrt{40}$

4 10

2 2 2 5

$\boxed{2i\sqrt{10}}$

Ex. $\sqrt{-125x^2}$

$i\sqrt{125(x\,x)}$

25 5

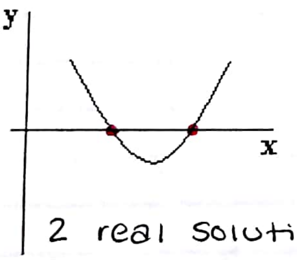
5 5

$\boxed{5x\,i\sqrt{5}}$

Where have we seen imaginary numbers before?

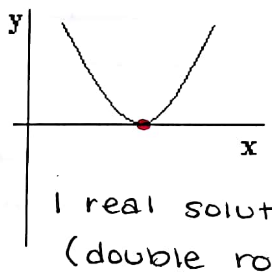
In Math 1, we learned the quadratic formula and the discriminant. The discriminant, $b^2 - 4ac$, determines the number of solutions and the type of solutions we will have with a quadratic equation.

Two Real
$b^2 - 4ac > 0$



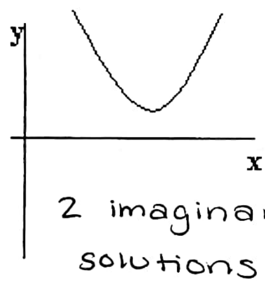
2 real solutions

One Real
$b^2 - 4ac = 0$



1 real solution (double root)

No Real
$b^2 - 4ac < 0$



2 imaginary solutions

Simplifying Powers of i's

To simplify i to any power, try to get the exponent to an even power by removing an i if the exponent is odd, and then reverse the "power to a power" rule by dividing by two. Simplify using the properties of algebra.

Ex. $i^{17} =$

$= i \cdot i^{16}$

$= i \cdot (i^2)^8$

$= i(-1)^8$

$= i(1)$

$= \boxed{i}$

Ex. $i^{98} =$

$= (i^2)^{49}$

$= (-1)^{49}$

$= \boxed{-1}$

Ex. $i^{39} =$

$= i \cdot i^{38}$

$= i \cdot (i^2)^{19}$

$= i \cdot (-1)^{19}$

$= i(-1)$

$= \boxed{-i}$

Ex. $i^{65} =$

$= i \cdot i^{64}$

$= i \cdot (i^2)^{32}$

$= i \cdot (-1)^{32}$

$= i(1)$

$= \boxed{i}$

Imaginary Numbers: For any positive b, $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$

Example 1: Simplify $2\sqrt{-12} \cdot 3\sqrt{-3}$.

$2i\sqrt{12} \cdot 3i\sqrt{+3}$

$6i^2\sqrt{36}$

$6i^2(6)$

$= 36i^2$

$= \boxed{-36}$

You Try! Simplify $\sqrt{-8} \cdot \sqrt{-32}$

$i\sqrt{8} \cdot i\sqrt{32}$

$i^2\sqrt{256}$

$16i^2 = \boxed{-16}$

Complex Numbers: What is a complex number?

a + bi

Real | imaginary

The real part is ALWAYS first!

Example 2: Name the real and imaginary part of $7 + 4i$

real imaginary

Adding and Subtracting Complex Numbers: Only combine like terms. Double check with your calculator.

a) Simplify $(6-4i)+(1+3i)$

$6 - 4i + 1 + 3i$

$\boxed{7 - i}$

b) Simplify $(4-6i)-(3-7i)$

$4 - 6i - 3 + 7i$

$\boxed{1 - i}$

Let x and y be real numbers. What are the values of x and y?

a) $(x+yi)-(7-3i) = 12+9i$

$x + yi - \cancel{7} + \cancel{3i} = 12 + 9i$

$+\cancel{7} \; -\cancel{3i} \quad +7 \; -3i$

$x + yi = 19 + 6i$

$\boxed{x = 19,\; y = 6}$

b) $(x+yi)+(9-4i) = -3-14i$

$x + yi + \cancel{9} - \cancel{4i} = -3 - 14i$

$-\cancel{9} \; +\cancel{4i} \quad -9 + 4i$

$x + yi = -12 - 10i$

$\boxed{x = -12,\; y = -10}$

Multiplying Complex Numbers: Make sure to FOIL. Double check with your calculator.

a) Simplify $(6-4i)(1+3i)$

$6 + 18i - 4i - 12i^2$

$6 + 14i + 12$

$\boxed{18 + 14i}$

b) Simplify $(4-6i)(3-7i)$

$12 - 28i - 18i + 42i^2$

$12 - 46i - 42$

$\boxed{-30 - 46i}$

Dividing Complex Numbers: Imaginary numbers may NEVER be in the denominator. To simplify, multiply the complex numbers by the conjugate (just like with radicals).

a) Simplify $\frac{3i}{2+4i} \cdot \frac{(2-4i)}{(2-4i)}$

$\frac{3i(2-4i)}{(2+4i)(2-4i)} = \frac{6i - 12i^2}{4 - 8i + 8i - 16i^2}$

$\frac{12 + 6i}{4 + 16} = \frac{12 + 6i}{20}$

$= \boxed{\frac{6 + 3i}{10}}$

b) Simplify $\frac{3i}{6-5i} \cdot \frac{(6+5i)}{(6+5i)}$

$\frac{3i(6+5i)}{(6-5i)(6+5i)} = \frac{18i + 15i^2}{36 - 30i + 30i - 25i^2}$

$= \frac{-15 + 18i}{36 + 25} = \boxed{\frac{-15 + 18i}{61}}$

Homework 3.2: Imaginary and Complex Numbers

Name: ______

Complete #1-8 all, and #10-24 even

Honors Math

Simplify the imaginary numbers. (No exponents!)

1) $(i)^{23}$

$\boxed{-i}$

2) $(i)^{113}$

$\boxed{i}$

Write the numbers using the imaginary unit, i.

3) $\sqrt{-15}$

$\boxed{i\sqrt{15}}$

4) $-\sqrt{-4}$

$\boxed{-2i}$

Simplify. Write your final answer in standard form.

5) $(-9 - i) - (-2 - 3i)$

$\boxed{-7 + 2i}$

6) $(8 + 3i) - (10 + i)$

$\boxed{-2 + 2i}$

7) $(-6 - 10i) - (-4 - i)$

$\boxed{-2 - 9i}$

8) $(-10 - 10i) + (-4 - 10i)$

$\boxed{-14 - 20i}$

Simplify.

9) $(8 - 2i)(5 + 2i)$

10) $(7 + i)(-4 + 3i)$

$\boxed{-31 + 17i}$

11) $(-3 + 7i)(6 + 5i)$

12) $(-8 + 8i)(-5 + 8i)$

$\boxed{-24 - 104i}$

13) $(1 - 4i)(8 + 4i)$

14) $4(3i)(5 + 7i)$

$\boxed{-84 + 60i}$

15) $(-2 - 2i)(-8 - i)$

16) $(7 - 6i)(-1 - 2i)$

$\boxed{-19 - 8i}$

More on the back!

Honors Math 3

3.3 Factoring Review

Unit 3

EQ: How can we polynomial expressions using different methods?

Factoring #1: Greatest Common Factor

All expressions have the potential of being factored using GCF. Check for it every time!

1. $3ab^2 - 6a^2b$

$\frac{3ab^2}{3ab} \quad \frac{6a^2b}{3ab}$

$\boxed{3ab(b-2a)}$

2. $5x^3 + 6xy$

$\frac{5x^3}{x} \quad \frac{6xy}{x}$

$\boxed{x(5x^2+6y)}$

3. $xyz + 3x^2y^2z^2$

$\frac{xyz}{xyz} \quad \frac{3x^2y^2z^2}{xyz}$

$\boxed{xyz(1+3xyz)}$

Factoring #2: Grouping (4-term polynomials)

Factor by grouping the first two terms together, the second two terms together, and removing a GCF.

4. $(30b^4 - 45b^3)(-10b^2 + 15b)$

$\frac{30b^4}{15b^3} \quad \frac{45b^3}{15b^3} \quad \frac{10b^2}{-5b} \quad \frac{15b}{-5b}$

$15b^3(2b-3) - 5b(2b-3)$

$(15b^3 - 5b)(2b-3)$

$\frac{15b^3}{5b} \quad \frac{5b}{5b}$

$\boxed{5b(3b^2-1)(2b-3)}$

5. $(6x^3 + 9x^2)(+2x + 3)$

$\frac{6x^3}{3x^2} \quad \frac{9x^2}{3x^2} \quad \frac{2x}{1} \quad \frac{3}{1}$

$3x^2(2x+3)\ 1(2x+3)$

$\boxed{(3x^2+1)(2x+3)}$

6. $(8t^3 + 36t^2)(+2t + 9)$

$4t^2(2t+9)\ 1(2t+9)$

$\boxed{(4t^2+1)(2t+9)}$

Factoring #3: Factoring trinomials ($ax^2 + bx + c$)

X-Factor (what multiplies to "ac" that adds to "b"), split into four terms, and continue by grouping.

7. $x^2 + 6x + 8$

Multiplies

8

4 2

6

Add

$(x^2+4x)(+2x+8)$

$x(x+4)+2(x+4)$

$\boxed{(x+2)(x+4)}$

8. $3x^2 - 18x + 24$

72

-6 -12

-18

$(3x^2-6x)(-12x+24)$

$3x(x-2) - 12(x-2)$

$(3x-12)(x-2)$

$\boxed{3(x-4)(x-2)}$

9. $2x^3 - 2x^2 - 12x$

-24

-6 4

-2

$(2x^3-6x^2)(4x^2-12x)$

$2x^2(x-3)\ 4x(x-3)$

$(2x^2+4x)(x-3)$

$\boxed{2x(x+2)(x-3)}$

Factoring #4: Difference of Squares $a^2 - b^2 = (a-b)(a+b)$

There must be a subtraction sign and two perfect square binomials in order for this to work!

10. $y^2 - \frac{9}{25}$

$\boxed{\left(y-\frac{3}{5}\right)\left(y+\frac{3}{5}\right)}$

11. $3x^2 - 75$

$3(x^2-25)$

$\boxed{3(x-5)(x+5)}$

12. $x^4 - 81$

$(x^2-9)(x^2+9)$

$\boxed{(x-3)(x+3)(x^2+9)}$

17) $\frac{-5-9i}{9+8i}$

18) $\frac{-4+10i}{3+4i}$

$\boxed{\frac{28+46i}{25}}$

19) $\frac{-5-3i}{7-10i}$

20) $\frac{-3-7i}{7+10i}$

$\boxed{\frac{-91-19i}{149}}$

21) $\frac{-1+i}{-5i}$

22) $\frac{-6-i}{i}$

$\boxed{-1+6i \quad \underline{\text{or}} \quad \frac{1-6i}{-1}}$

23) $\frac{2+5i}{-i}$

24) $\frac{-4-4i}{4i}$

$\boxed{\frac{1-i}{-1} \quad \underline{\text{or}} \quad -1+i}$

Answers:

1. $-i$
2. i
3. $i\sqrt{15}$
4. $-2i$
5. $-7+2i$
6. $-2+2i$
7. $-2-9i$
8. $14-20i$

10. $-31+17i$

12. $-24-104i$

14. $-84+60i$

16. $-19-8i$

18. $\frac{28-46i}{25}$

20. $\frac{-91-19i}{149}$

22. $-1+6i$

24. $-1+i$

3.3 Factoring Review

SWBAT factor expressions using sum and difference of cubes.

Sum of Cubes	Difference of Cubes
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a = \sqrt[3]{a^3}$

$b = \sqrt[3]{b^3}$

Just remember to use SOAP

Same – Opposite – Always Positive

(a	b)	(a^2	ab	b^2)

$\sqrt[3]{27x^3} = 3x$

$\sqrt[3]{y^3} = y$

a) $x^3 - 1$

a	b	a^2	ab	b^2
x	1	x^2	1x	1

$\boxed{(x - 1)(x^2 + 1x + 1)}$

same opposite Always Positive

b) $x^3 + y^3$

a	b	a^2	ab	b^2
x	y	x^2	xy	y^2

$\boxed{(x + y)(x^2 - xy + y^2)}$

c) $27x^3 - y^3$

a	b	a^2	ab	b^2
3x	y	$9x^2$	3xy	y^2

$\boxed{(3x - y)(9x^2 - 3xy + y^2)}$

d) $m^3 - 216$

a	b	a^2	ab	b^2
m	6	m^2	6m	36

$\boxed{(m - 6)(m^2 + 6m + 36)}$

e) $27 - y^3$

a	b	a^2	ab	b^2
3	y	9	3y	y^2

$\boxed{(3 - y)(9 + 3y + y^2)}$

f) $125x^3 + 8a^3$

a	b	a^2	ab	b^2
5x	2a	$25x^2$	10ax	$4a^2$

$\boxed{(5x + 2a)(25x^2 - 10ax + 4a^2)}$

g) $1000 + 27a^3$

$\boxed{(10 + 3a)(100 - 30a + 9a^2)}$

h) $s^3 - 64$

$\boxed{(s - 4)(s^2 + 4s + 16)}$

i) $y^3 + 125$

$\boxed{(y + 5)(y^2 - 5y + 25)}$

Homework 3.3: Factoring Review

Name: ____________________

Directions: Complete the evens in each column. Show work on a separate sheet of paper.

Honors Math

GCF		Grouping	
1. $3a + 6$	1. $3(a+2)$	1. $5x + 15 + xy + 3y$	1. $(y+5)(x+3)$
2. $4x - 20$	2. $4(x-5)$	2. $xy + y + 2x + 2$	2. $(y+2)(x+1)$
3. $2y^3 + 8xy$	3. $2y(y^2+4x)$	3. $2y - 8 + xy - 4x$	3. $(x+2)(y-4)$
4. $5x + 10y - 15$ 4. ________	4. $5(x+2y-3)$	4. $6x - 42 + xy - 7y$	$(y+6)(x-7)$
5. $42m - 7$	5. $7(6m-1)$	5. $3xy - 6x + 8y - 16$	5. $(3x+8)(y-2)$
6. $18xy^2 + 6x^3 - 12x^2$	6. $6x(3y^2+x^2-2x)$	6. $xy - 2yz + 5x - 10z$	6. $(y+5)(x-2z)$
7. $7a + 21p + 14$	7. $7(a+3p+2)$	7. $y^3 + 3y^2 + y + 3$	7. $(y^2+1)(y+3)$
8. $40x^8y^6 - 16x^9y^5$	8. $8x^8y^5(5y-2x)$	8. $x^3 + 4x + x^2 + 4$	8. $(x+1)(x^2+4)$
9. $x(y+3) + 5(y+3)$	9. $(x+5)(y+3)$	9. $5xy + 15x + 6y + 18$	9. $(5x+6)(y+3)$
10. $12x^3 + 16x^2 - 8x$	10. $4x(3x^2+4x-2)$	10. $2x^3 + x^2 + 8x + 4$	10. $(x^2+4)(2$[illegible]$1)$
11. $2y^2 - 10y + 20$	11. $2(y^2-5y+10)$	11. $4x^2 - 8xy - 3x + 6y$	11. $(4x-3)(x-2y)$
12. $24x - 16$	12. $8(3x-2)$	12. $2x^3 - x^2 - 10x + 5$	12. $(x^2-5)(2x-1)$
13. $20xyz + 12x^2z - 40yz$ 13. ________	13. $4z(5xy+3x^2-10y)$	13. $y^2 - 3y + yz - 3z$	$(y-z)(y-3)$
14. $a^5 + 3a^4 - 6a^3 + 9a^2$	14. $a^2(a^3+3a^2-6a+9)$	14. $5x^2 - 20x^2y + 5z - 20yz$	14. $5(x^2+z)(1-4y)$
15. $y^7 - y^2$	15. $y^2(y^5-1)$	15. $2x - xy + 18 - 9y$	15. $-1(y-2)(x+9)$
16. $6t^2 + 24$	16. $6(t^2+4)$	16. $12x + 10 + 6xy + 5y$	16. $(y+2)(6x+5)$
17. $-5x^3 + 10x^2$	17. $-5x^2(x-2)$	17. $7y - 7 + 5xy - 5x$	17. $(5x+7)(y-1)$
18. $-9a^2b + 18a^2b^2 - 3ab$	18. $-3ab(3a-6ab+1)$	18. $6x^2y - 21x^2 - 4y + 14$	18. $(3x^2-2)(2y-7)$
19. $25x^4z + 15x^3z + 5x^2z$	19. $5x^2z(5x^2+3x+1)$	19. $30 + 5y^2 - 6x - xy^2$	19. $-1(x-5)(y^2+6)$
20. $3y^2 + 5x$	20. prime	20. $4ax - 4ab - 2bx + 2b^2$	20. $2(2a-b)($[illegible]$b)$

Binomials		Trinomials	
1. x^2-4	1. $(x-2)(x+2)$	1. x^2+7x+6	1. $(x+6)(x+1)$
2. y^2-36	2. $(y-6)(y+6)$	2. x^2+6x+8	2. $(x+4)(x+2)$
3. $100-p^2$	3. $(10-p)(10+p)$	3. $x^2+13x+30$	3. $(x+10)(x+3)$
4. $4x^2-1$	4. $(2x-1)(2x+1)$	4. $x^2+10x+25$	4. $(x+5)^2$
5. $9t^2-1$	5. $(3t-1)(3t+1)$	5. $x^2-8x+15$	5. $(x-5)(x-3)$
6. a^2+25	6. prime	6. x^2-6x+9	6. $(x-3)^2$
7. $49x^2-16$	7. $(7x-4)(7x+4)$	7. $x^2-10x+9$	7. $(x-9)(x-1)$
8. $4y^2-25$	8. $(2x-5)(2x+5)$	8. $x^2-3x-18$	8. $(x-6)(x+3)$
9. $12x^2-27$	9. $3(2x+3)(2x-3)$	9. x^2-x-30	9. $(x-6)(x+5)$
10. $9z^2-36$	10. $9(z+2)(z-2)$	10. x^2-x-2	10. $(x-2)(x+1)$
11. a^3-27	11. $(a+3)(a^2-3a+9)$	11. x^2+x-42	11. $(x-6)(x+7)$
12. $8y^3+1$	12. $(2y+1)(4y^2-2y+1)$	12. $y^2+4y-12$	12. $(y+6)(y-2)$
13. y^3+125	13. $(y+5)(y^2-5y+25)$	13. $2a^2-9a-5$	13. $(2a+1)(a-5)$
14. b^3+8	14. $(b+2)(b^2-2b+4)$	14. $3c^2+8c+4$	14. $(3c+2)(c+2)$
15. x^3+64	15. $(x+4)(x^2-4x+16)$	15. $2x^2+7x+5$	15. $(2x+5)(x+1)$
16. y^3-1	16. $(y-1)(y^2+y+1)$	16. $6y^2-11y-10$	16. $(3y+2)(2y-5)$
17. $27a^3-8$	17. $(3a-2)(9a^2+6a+4)$	17. $4a^2-8a-21$	17. $(2a+3)(2a-7)$
18. c^3-125	18. $(c-5)(c^2+5c+25)$	18. $3x^2+x-2$	18. $(3x-2)(x+1)$
19. $8x^3-27$	19. $(2x-3)(4x^2+6x+9)$	19. $3x^2-5x+1$	19. prime
20. $64y^3-1$	20. $(4y-1)(16y^2+4y+1)$	20. $8y^2-22y+5$	20. $(4y-1)(2y-5)$

Honors Math 3

3.4 Solving Quadratics

EQ: What methods can we use to solve quadratic equations and verbal expressions?

SOLVE BY FACTORING!

Set all equations equal to zero before beginning!

Greatest Common Factor
$ax + bx = x(a + b)$

Difference of Squares
$x^2 - y^2 = (x + y)(x - y)$

Factoring Trinomials
(X-Factor)
$ax^2 + bx + c$

Grouping
Four-term polynomials

Step 1: Factor Away! → **Step 2:** Set each factor = 0 → **Step 3:** Solve for x.

Directions: Solve each of the following by factoring. Check your solutions by graphing.

1. $(2x + 1)(3x - 4) = 0$

$2x+1=0 \qquad 3x-4=0$

$2x=-1 \qquad 3x=4$

$x=-\frac{1}{2} \qquad x=\frac{4}{3}$

$\{-\frac{1}{2}, \frac{4}{3}\}$

2. $x(3x + 9) = 0$

$x=0 \qquad 3x+9=0$

$3x=-9$

$x=-3$

$\{0, -3\}$

3. $-x^2 = -121$

$-1(-x^2+121)=0$

$x^2-121=0$

$(x-11)(x+11)=0$

$x-11=0 \qquad x+11=0$

$x=11 \qquad x=-11$

$\{-11, 11\}$

4. $5x^2 + 32x = -28x$

$5x^2+60x=0$

$5x(x+12)=0$

$5x=0 \qquad x+12=0$

$x=0 \qquad x=-12$

$\{0, -12\}$

5. $45x^2 + 56x = -16$

720; 20, 36; 56

$45x^2+56x+16=0$

$(45x^2+20x)(+36x+16)=0$

$5x(9x+4)4(9x+4)=0$

$5x+4=0 \qquad 9x+4=0$

$5x=-4 \qquad 9x=-4$

$x=-4/5 \qquad x=-4/9$

$\{-\frac{4}{5}, -\frac{4}{9}\}$

6. $3x^2 + 8x + 5 = 0$

15; 5, 3; 8

$(3x^2+3x)(+5x+5)=0$

$3x(x+1)5(x+1)=0$

$3x+5=0 \qquad x+1=0$

$3x=-5 \qquad x=-1$

$x=-\frac{5}{3}$

$\{-1, -\frac{5}{3}\}$

7. The product of two consecutive negative integers is 1122. What are the numbers?

$1^{st}: x$

$2^{nd}: x+1$

$x(x+1)=1122$

$x^2+x=1122$

$x^2+x-1122=0$

1122; 34, −33; 1

$(x+34)(x-33)=0$

$x+34=0 \qquad x-33=0$

$x=-34 \qquad \cancel{x=33}$

not negative

$\boxed{-34, -33}$

8. The width of a rectangle is (x+1) and the length is (x-6). What is the length and width of the rectangle if the area is 30 square feet?

$(x+1)(x-6)=30$

$x^2-6x+x-6=30$

$x^2-5x-6=30$

$x^2-5x-36=0$

−36; −9, 4; −5

$(x-9)(x+4)=0$

$x-9=0 \qquad x+4=0$

$x=9 \qquad x=-4$

Width cannot be negative

Width $= 9+1 = \boxed{10ft}$

Length $= 9-6 = \boxed{3ft}$

9. The area of a triangular lot is 225 square feet. The base of the lot is 7 more than its height. Find the length of the base and the height.

base $= h+7$

height $= h$

$A=\frac{bh}{2}$

$\frac{h(h+7)}{2}=225$

$h^2+7h=450$

$h^2+7h-450=0$

−450; 25, −18; 7

$(h+25)(h-18)=0$

$h+25=0 \qquad h-18=0$

$h=-25 \qquad h=18$

height cannot be negative

$\boxed{\text{base} = 25 \quad \text{height} = 18}$

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The Quadratic Formula

$$X = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Step 1: Set the equation equal to 0.

Step 2: Label the "a", "b" and "c" terms.

Step 3: Substitute each value into the discriminant.

Step 4: Substitute back into formula and simplify.

Step 5: Split the equation into the + and - solutions

Step 6: Solve for x.

Using the Quadratic Formula

What are the roots of the equation $2x^2 - 4x + 7 = 0$? Use the quadratic formula to solve.

$a = 2$
$b = -4$
$c = 7$

$$X = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(7)}}{2(2)}$$

$$X = \frac{4 \pm \sqrt{-40}}{4}$$

$$X = \frac{4 \pm i\sqrt{40}}{4}$$

40 → 4, 10; 4 → 2, 2; 10 → 2, 5

$$X = \frac{4 \pm 2i\sqrt{10}}{4}$$

← Don't divide the radicand! We already simplified it!

$$X = \frac{2 \pm i\sqrt{10}}{4}$$

$$\left\{ \frac{2 + i\sqrt{10}}{2}, \frac{2 - i\sqrt{10}}{2} \right\}$$

What are the solutions of $x^2 - 34x + 289 = 0$? Use the quadratic formula to solve.

$a = 1$
$b = -34$
$c = 289$

$$X = \frac{34 \pm \sqrt{(-34)^2 - 4(1)(289)}}{2(1)}$$

$$X = \frac{34 \pm \sqrt{0}}{2}$$

$$X = \frac{34 \pm 0}{2}$$

$$X = \frac{34+0}{2} \qquad X = \frac{34-0}{2}$$

$$X = 17 \qquad X = 17$$

$\{17\}$

What are the roots of the equation $x^2 - 8x = 33$? Use the quadratic formula to solve.

$x^2 - 8x - 33 = 0$

$a = 1$
$b = -8$
$c = -33$

$$X = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-33)}}{2(1)}$$

$$X = \frac{8 \pm \sqrt{196}}{2}$$

$$X = \frac{8 \pm 14}{2}$$

$$X = \frac{8+14}{2} \qquad X = \frac{8-14}{2}$$

$$X = 11 \qquad X = -3$$

$\{-3, 11\}$

Homework 3.4: Solving Quadratics

Name: ______________________

Honors Math

Directions: Solve #2-24 even, #25, 26, 27.

Solve using the Quadratic Formula - Level 2

1) $n^2 + 9n + 11 = 0$

2) $5p^2 - 125 = 0$ $\{\pm 5\}$

3) $m^2 + 5m + 6 = 0$

4) $2x^2 - 4x - 30 = 0$ $\{5, -3\}$

Solve using the Quadratic Formula - Level 3

5) $b^2 - 12b + 10 = -10$ $\{10, 2\}$

6) $6r^2 - 5r - 4 = 7$ $\{11/6, -1\}$

7) $7x^2 - 16 = 6$

8) $6n^2 - 10n - 16 = 3$ $\left\{\frac{5 \pm \sqrt{139}}{6}\right\}$

Solve using the Quadratic Formula - Level 4

9) $4a^2 - 22 = -10a$ $\frac{-5 \pm \sqrt{113}}{4}$

10) $n^2 - 45 = 12n$ $\{15, -3\}$

11) $5v^2 - 2 - v = -v$

12) $4x^2 - 5x - 3 = 2x^2$ $\{3, -1/2\}$

Solve by Factoring - Level 2

13) $p^2 + 6p + 5 = 0$ $\{-1, -5\}$

14) $k^2 - 8k = 0$ $\{0, 8\}$

15) $x^2 - 7x = 0$

16) $a^2 + 5a = 0$ $\{0, -5\}$

Solve by Factoring - Level 3

17) $6n^2 + 5n - 25 = 0$ $\{\frac{5}{3}, -\frac{5}{2}\}$

18) $2x^2 - 11x - 21 = 0$ $\{-3/2, 7\}$

19) $10r^2 + 75r + 140 = 0$

20) $60m^2 + 4m - 160 = 0$ $\{8/5, -5/3\}$

Solve by Factoring - Level 4

21) $4x^2 - 17x + 10 = -5$

22) $2n^2 + 13n + 19 = 4$ $\{-3/2, -5\}$

23) $5v^2 + 3 = -16v$

24) $20b^2 - 40b = 25$ $\{5/2, -1/2\}$

25. The larger leg of aright triangle is 7 cm longer than its smaller leg. The hypotenuse is 8 cm longer than the smaller leg. How many centimeters long is the smaller leg?

$\boxed{5\text{cm}}$

26. The area of a rectangular floor is 105 square feet. If its length is 1 more than twice its width, find the length and width of the floor.

w = 7

l = 15

27. What is the smallest of 3 consecutive positive integers if the product of the smaller two integers is six le[ss] than 6 times the largest?

6

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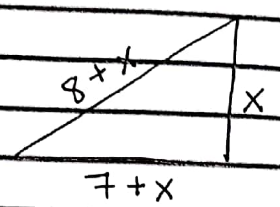
25)



$(7+x)^2 + x^2 = (8+x)^2$

$x^2 + 14x + 49 + x^2 = x^2 + 16x + 64$

$2x^2 + 14x + 49 = x^2 + 16x + 64$

$x^2 - 2x - 15 = 0$

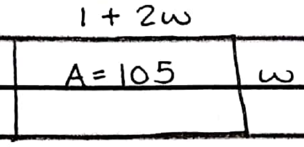
$(x+5)(x-3) = 0$

-15, 3, 2, -5

$x = +5 \quad x = -3$

cannot be negative length

26)



-105, 2

$(1+2w)w = 105$

$2w^2 + 2w = 105$

$2w^2 + 2w - 105 = 0$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{(2)^2-4(2)(-105)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{844}}{4} = \frac{-2 \pm 2\sqrt{211}}{4}$$

$$= \frac{-1 \pm \sqrt{211}}{2} = \boxed{6.763} \quad -7.763$$

cannot be negative length

$w = 6.763$

$l = 14.526$

27)

1st: x

2nd: $x+1$

3rd: $x+2$

$(x)(x+1) = 6(x+2) - 6$

$x^2 + x = 6x + 12 - 6$

$x^2 - 5x = 6$

$x^2 - 5x - 6 = 0$

-6, 1, -6, -5

$(x^2 + x)(-6x - 6) = 0$

$x(x+1) - 6(x+1) = 0$

$(x-6)(x+1) = 0$

$x = 6 \quad x = -1$

needs to be positive

sma

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