

# SOLVE RADICAL Equations

## Radicals on one side

If two expressions are equal, then their squares are equal.

$$\text{If } a = b, \\ \text{then } a^2 = b^2$$

$$\text{If } \sqrt{x} = 5, \\ \text{then } (\sqrt{x})^2 = 5^2$$

Example 1:  $\sqrt{x} + 3 = 10$   
 $\quad \quad \quad -3 \quad \quad -3$

$$\sqrt{x} = 7 \\ (\sqrt{x})^2 = (7)^2 \\ \boxed{x = 49}$$

Example 2:  $-3\sqrt{x} = -18$   
 $\quad \quad \quad -3 \quad \quad \quad -3$

$$\sqrt{x} = 6 \\ (\sqrt{x})^2 = (6)^2 \\ \boxed{x = 36}$$

You Try!  $2\sqrt{x} - 5 = -1$   
 $\quad \quad \quad +5 \quad \quad +5$

$$\frac{2\sqrt{x}}{2} = \frac{4}{2}$$

$$\sqrt{x} = 2 \\ (\sqrt{x})^2 = (2)^2 \\ \boxed{x = 4}$$

Radicals on both sides

Example 1:  $\sqrt{x+4} = \sqrt{2x-1}$

$$(\sqrt{x+4})^2 = (\sqrt{2x-1})^2$$

$$\begin{array}{r} x+4 = 2x-1 \\ -x \quad -x \end{array}$$

$$\begin{array}{r} 4 = x-1 \\ +1 \quad +1 \end{array}$$

$$\boxed{5 = x}$$

You Try!  $\sqrt{3x+8} = \sqrt{x+4}$

$$(\sqrt{3x+8})^2 = (\sqrt{x+4})^2$$

$$\begin{array}{r} 3x+8 = x+4 \\ -x \quad -x \end{array}$$

$$\begin{array}{r} 2x+8 = 4 \\ -8 \quad -8 \end{array}$$

$$\frac{2x}{2} = \frac{-4}{2}$$

$$\boxed{x = -2}$$

### Extraneous Solutions

Example 1: Solve the equation. Check for extraneous solutions

$$x = \sqrt{42-x}$$

CHECK

$$(x)^2 = (\sqrt{42-x})^2$$

$$x^2 = 42-x$$

$$x^2+x-42=0$$

$$(x-6)(x+7)=0$$

$$x-6=0 \quad x+7=0$$

$$x=6 \quad x=-7$$

$$x=6$$

$$6 = \sqrt{42-6}$$

$$6 = \sqrt{36}$$

$$6 = 6$$

✓

$$x = -7$$

$$-7 = \sqrt{42-(-7)}$$

$$-7 = \sqrt{49}$$

$$-7 \neq 7$$

You Try! Solve the equation. Check for extraneous solutions

$$\sqrt{2-x} = x+4$$

$$(\sqrt{2-x})^2 = (x+4)^2$$

$$2-x = (x+4)(x+4)$$

$$2-x = x^2+4x+4x+16$$

$$2-x = x^2+8x+16$$

$$\begin{array}{r} -2+x \\ +x \end{array} \quad \begin{array}{r} +x \\ -2 \end{array}$$

$$0 = x^2 + 9x + 14$$

$$0 = (x+7)(x+2)$$

$$x+7=0 \quad x+2=0$$

$$x=-7 \quad x=-2$$

CHECK

$$x = -7$$

$$\sqrt{2-(-7)} = (-7)+4$$

$$\sqrt{9} = -3$$

$$3 \neq -3$$

$$x = -2$$

$$\sqrt{2-(-2)} = (-2)+4$$

$$\sqrt{4} = 2$$

$$2 = 2$$

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