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Unit 5 Study Guide

Make sure to study all parts included in this study guide. This unit has a lot of material so make sure to look back at all your notes and on my website: msbluemath.weebly.com

Exponent Properties

Multiplying exponents with the same base (Product Rule) – Add the exponents, base stays the same

Dividing exponents with same base (Quotient Property) – Subtract the exponents, base stays the same

Power to a power – Multiply the exponents, base stays the same

Negative exponents – Take the reciprocal, keep the base and make the exponent positive

Zero power – Any number, except 0, raised to the zero power is equal to 1

1) $2x^3 \cdot 2x^3$
$4x^6$

2) $4x^3y^2 \cdot 3x^{-4}y^{-3}$
$12x^{-1}y^{-1} = \frac{12}{xy}$

3) $(4x^3)^2$
$4^2x^6 = 16x^6$

4) $(x^2y^{-1})^2$
$x^4y^{-2} = \frac{x^4}{y^2}$

5) $\frac{x^2}{2x^3}$
$\frac{1}{2x}$

6) $\frac{3x^4}{3x^3}$
$\frac{1x^1}{1} = x$

7) $\frac{2x^4y^{-4}z^{-3}}{3x^2y^{-3}z^4} = \frac{2x^2y^{-1}z^{-7}}{3} = \frac{2x^2}{3yz^7}$

Reducing Radicals

The inverse of squaring is finding a square root.

Definition of square root – If $x^2 = y$, then x is a square root of y.

If the exponent is **even**, there is a **positive (+)** and **negative (-)** root.

If the exponent is **odd**, there is only **one root.**

Radicals can be written as exponents. Taking the square root of a number is the same as raising the number to the ½ power.

$$\sqrt{x} = x^{\frac{1}{2}} \quad \sqrt[3]{x} = x^{\frac{1}{3}} \quad \sqrt[4]{x} = x^{\frac{1}{4}}$$

Simplifying Radicals Steps:

- Find the prime factorization of the expression inside the radical (break it down)
- Determine the index of the radical (square root = 2, cube root = 3, etc.)
- Move each group of numbers outside of the radical that match the index. (If you have a cube root, your numbers/variables need to be in groups of 3)
- Simplify expressions inside and outside of the radical

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1) $\sqrt{150}$ $= \pm 5\sqrt{6}$

2) $\sqrt{48}$ $= \pm 4\sqrt{3}$

3) $\sqrt[3]{27}$ $= 3$

4) $\sqrt{64x^4}$ $= 8x^2$

5) $\sqrt[4]{1215x^5y^8}$ $\longrightarrow \pm 3xy^2\sqrt[4]{15x}$

6) $\sqrt{1500x^6y^5}$ $\pm 10x^3y^2\sqrt{15y}$

Write the standard form to vertex form

- Step 1: Put parenthesis around your x terms
- Step 2: Divide your b term by 2 and square it
- Step 3: Add this number inside your parentheses and subtract it from the constant on the outside
- Step 4: Factor the parenthesis by dividing b by 2

1) $(p^2 + 14p) - 38 - 49$

$\frac{14}{2} = 7 \rightarrow 7^2 = 49$

$\boxed{(p+7)^2 - 87}$

2) $(a^2 + 14a) - 51 - 49$

$\frac{14}{2} = 7 \quad 7^2 = 49$

$\boxed{(x+7)^2 - 100}$

3) $(v^2 + 6v) - 59 - 9$

$\frac{6}{2} = 3 \quad 3^2 = 9$

$\boxed{(v+3)^2 - 68}$

Use the Quadratic Formula to solve

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $x^2 - 24x - 19 = 41$

$-41 \quad -41$

$x^2 - 24x - 60 = 0$

$\frac{-(-24) \pm \sqrt{(-24)^2 - 4(1)(-60)}}{2(1)}$

$\frac{24 \pm \sqrt{816}}{2} = \frac{24 \pm 4\sqrt{51}}{2}$

$\boxed{12 \pm 2\sqrt{51}}$

2) $6x^2 + 5x = 2$

$-2 \quad -2$

$6x^2 + 5x - 2 = 0$

$\frac{-(5) \pm \sqrt{(5)^2 - 4(6)(-2)}}{2(6)} = \boxed{\frac{-5 \pm \sqrt{73}}{12}}$

3) $6x - 5 = -x^2$

$+x^2 \quad +x^2$

$x^2 + 6x - 5 = 0$

$\frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-5)}}{2(1)}$

$\frac{-6 \pm \sqrt{56}}{2}$

$\frac{-6 \pm 2\sqrt{14}}{2}$

$\boxed{-3 \pm \sqrt{14}}$

4) $x^2 - 28x = -120$

$+120 \quad +120$

$x^2 - 28x + 120 = 0$

$\frac{-(-28) \pm \sqrt{(-28)^2 - 4(1)(120)}}{2(1)}$

$\frac{28 \pm \sqrt{304}}{2} = \frac{28 \pm 4\sqrt{19}}{2}$

$\boxed{14 \pm 2\sqrt{19}}$

5) $12x^2 + 48x - 11 = 13$

$-13 \quad -13$

$12x^2 + 48x - 24 = 0$

$\frac{-(48) \pm \sqrt{(48)^2 - 4(12)(-24)}}{2(12)} = \frac{-48 \pm \sqrt{3456}}{24}$

$\frac{-48 \pm 24\sqrt{6}}{24} = \boxed{-2 \pm \sqrt{6}}$

$2\sqrt{28}$ $2\sqrt{14}$ $7\sqrt{7}$ 1

Unit 5 Study Guide $\frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-3)}}{2(2)}$

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$2x^2 - 2x - 3 = 0$ **6)** $2x(x-1) = 3$ $\quad \frac{2 \pm \sqrt{28}}{4} = \frac{2 \pm 2\sqrt{7}}{4} = \boxed{\frac{1 \pm \sqrt{7}}{2}}$

$2x^2 - 2x = 3$
$-3 \quad -3$

Solve the quadratic by factoring

Zero-product property – If (a)(b) = 0, then a = 0 or b = 0

Steps to solve by factoring:

- Move all of the terms to one side of the equation, making the other side zero
- Factor, or rewrite the other side of the equation as multiplication
- Apply the zero-product property

1) $(k+1)(k-5) = 0$

$k+1=0 \quad k-5=0$
$-1 \quad -1 \quad +5 \quad +5$
$\boxed{k=-1} \quad \boxed{k=5}$

$x^2 - 11x + 24 \leftarrow$ **2)** $x^2 - 11x + 19 = -5$
$+5 \quad +5$

c|b: 24|-11

$x-3=0 \quad x-8=0 \quad (x-3)(x-8)=0$

$x=3 \quad x=8$

3) $6n^2 - 18n - 18 = 6$
$-6 \quad -6$

1, 24; 2, 12; (-3, 8)

$n+1=0 \quad n-4=0$

$n=-1 \quad n=4$

$6n^2 - 18n - 24 = 0$

$-4 | -3$; (1, -4)

$6(n^2 - 3n - 4) = 0$

$(n+1)(n-4) = 0$

c|b: -35 | 2; 1, 35; (5, 7)

4) $b^2 + 5b - 35 = 3b \longrightarrow b^2 + 2b - 35 = 0$
$-3b \quad -3b$

$(x-5)(x+7) = 0 \quad x-5=0 \quad x+7=0$

$\boxed{x=5} \quad \boxed{x=-7}$

5) $5r^2 - 44r + 120 = -30 + 11r$

6) $-4k^2 - 8k - 3 = -3 - 5k^2$
$+5k^2 \quad +3 \quad +3 \quad +5k^2$

$k^2 - 8k = 0 \quad k(k-8) = 0$

$k=0 \quad k-8=0$

$k=8$

Determine the nature of the roots by finding the discriminant

The **discriminant** can tell us **how many** and **what kind** of roots our quadratic has

Formula: $b^2 - 4ac$

If $b^2 - 4ac$ is greater than zero, then you will have 2 real roots

If $b^2 - 4ac$ is equal to zero, then you will have 1 real roots

If $b^2 - 4ac$ is less than zero, then you will have 2 imaginary roots

a b c

1) $-2x^2 - 8x - 14 = -6$

$(-8)^2 - 4(-2)(-14) = -48$

$-48 < 0$ 2 imaginary roots

2) $-10n^2 - 3n - 9 = -2n$
$+2n \quad +2n$

$-10n^2 - n - 9 = 0$

$(-1)^2 - 4(-10)(-9) = -359$

2 imaginary roots

3) $4a^2 = 8a - 4$
$-8a \quad -8a \quad +4$
$+4$

$4a^2 - 8a + 4 = 0$

$(-8)^2 - 4(4)(4) = 0$

1 real root

Imaginary Numbers

An **imaginary number** gives us the ability to take the **square root** of a **negative number**

$$i = \sqrt{-1}$$

Other important imaginary numbers

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$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

1) $\sqrt{-25} = \pm i\sqrt{25} = \boxed{\pm 5i}$

2) $\sqrt{-16} = \pm i\sqrt{16} = \boxed{\pm 4i}$

3) $\sqrt{-144} = \pm i\sqrt{144} = \boxed{\pm 12i}$

Complex Numbers

Complex Conjugates – $a + bi$ and $a - bi$

When multiplied, complex conjugates will always be a **real number**

Adding & Subtracting Complex Numbers – Add/subtract their real parts and their imaginary parts separately

Multiplying Complex Numbers – Multiply using the distributive property (FOIL)

Dividing Complex Numbers – To divide, multiply top and bottom on the complex conjugate of denominators

Simplify:

1) $\sqrt{-64} = \pm i\sqrt{64} = \boxed{\pm 8i}$

2) $\sqrt{-20} = \pm i\sqrt{20} = \boxed{\pm 2i\sqrt{5}}$

2|20, 2|10, 5|5, 1

Solve:

1) $\sqrt{x^2} = \sqrt{-50}$

$x = \sqrt{-50}$

$x = \pm i\sqrt{50}$

$\boxed{x = \pm 5i\sqrt{2}}$

2|50, 5|25, 5|5, 1

2) $x^2 - 7 = -82$

$+7 \quad +7$

$\sqrt{x^2} = \sqrt{-75}$

$x = \sqrt{-75}$

$x = \pm i\sqrt{75}$

$\boxed{x = \pm 5i\sqrt{3}}$

3|75, 5|25, 5|5, 1

3) $-3x^2 + 17 = 113$

$-17 \quad -17$

$\frac{-3x^2}{-3} = \frac{96}{-3}$

$\sqrt{x^2} = \sqrt{-32}$

$x = \sqrt{-32}$

$x = \pm i\sqrt{32}$

$\boxed{x = \pm 4i\sqrt{2}}$

2|32, 2|16, 2|8, 2|4, 2|2

Simplify by adding/subtracting/multiplying/dividing:

1) $(-7i) + (8 + i) + (-4 + 8i)$

$0+8 = 8 \qquad -7i + i = -6i$

$(8 - 6i) + (-4 + 8i)$

$8 + -4 = 4 \qquad -6i + 8i = 2i$

$\boxed{(4 + 2i)}$

2) $(7 - 4i) - (-8 - 4i) + (-6 + 8i)$

$7 - -8 = 15 \qquad -4i - -4i = 0i$

$(15 + 0i) + (-6 + 8i)$

$15 + -6 = 9 \qquad 0i + 8i = 8i$

$\boxed{(9 + 8i)}$

3) $(5 - 3i)(-8 + 5i)$

$-40 + 25i + 24i - 15i^2 \leftarrow (-1)$

$-40 + 49i + 15$

$\boxed{-25 + 49i}$

4) $\frac{i}{2-3i} \cdot \frac{(2+3i)}{(2+3i)} = \frac{i(2+3i)}{(2-3i)(2+3i)} = \frac{2i + 3i^2}{4 + 6i - 6i - 9i^2}$

$= \frac{2i + 3(-1)}{4 - 9(-1)} = \frac{-3 + 2i}{13}$

$\boxed{= \frac{-3}{13} + \frac{2i}{13}}$